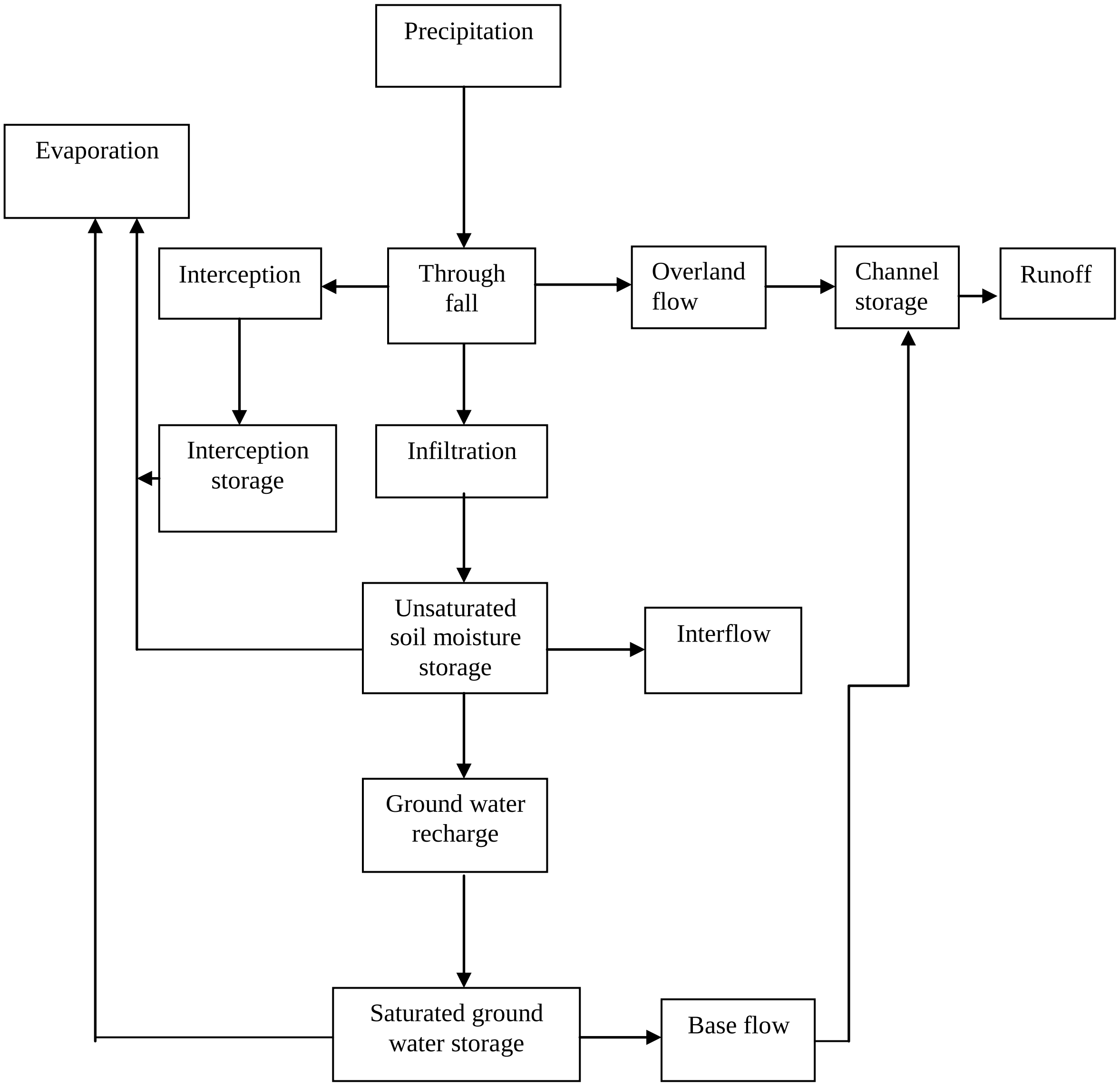
Ground water

- Porous media characteristics
- Flow in porous media (Darcy eq.)
- Aquifer system
- General flow equation and its solution
- Advection dispersion relations
- Control methods of some ground water pollution situation

Groundwater: is water stored under the surface of the ground in the tiny pore spaces between rock, sand, soil, and gravel. It occurs in two “zones”: an upper, unsaturated zone where most of the pore spaces are filled with air, and a deeper, saturated zone in which all the pore spaces are filled with water

How Does Groundwater Move

Ground water is part of hydrological cycle , the endless circulation of water between ocean atmosphere land is called hydrological cycle which can be representative in the following flowchart.



Ground water is more than a resource , it is an important feature of the natural environmental because , it leads to:

1. environmental problems through ground water contamination by:
   - Waste disposal technique (sanitary landfill for solid waste and deep well injection for liquid waste).
   - Subsurface pollution can be caused by leakage from ponds and lagoons (which are widely used as components of larger waste – disposal systems).
   - Leaching of animal waste , fertilizers and pesticide from agricultural soils.
2. Ground water contributors to geotechnical problems as slope stability and land subsidence through withdrawals from subsurface aquifers
3. Ground water is a key to understanding a wide variety of earthquakes, the migration and accumulation of petroleum.

- The most geologic role played by ground water lies in the control that fluid pressures exert on the mechanisms of faulting and thrusting.
- All rocks that underline the earth's surface can be classified either as aquifers or as confined beds.
- Aquifer: is a rock unit that will yield water in a usable quantity to a well or spring or a permeable water bearing bed or layer that hold and transmit water.
- Confined bed : is a rock unit having very low hydraulic conductivity that restricts the movement of ground water either into or out of adjacent aquifers.

Ground water occurs in aquifers under two different conditions

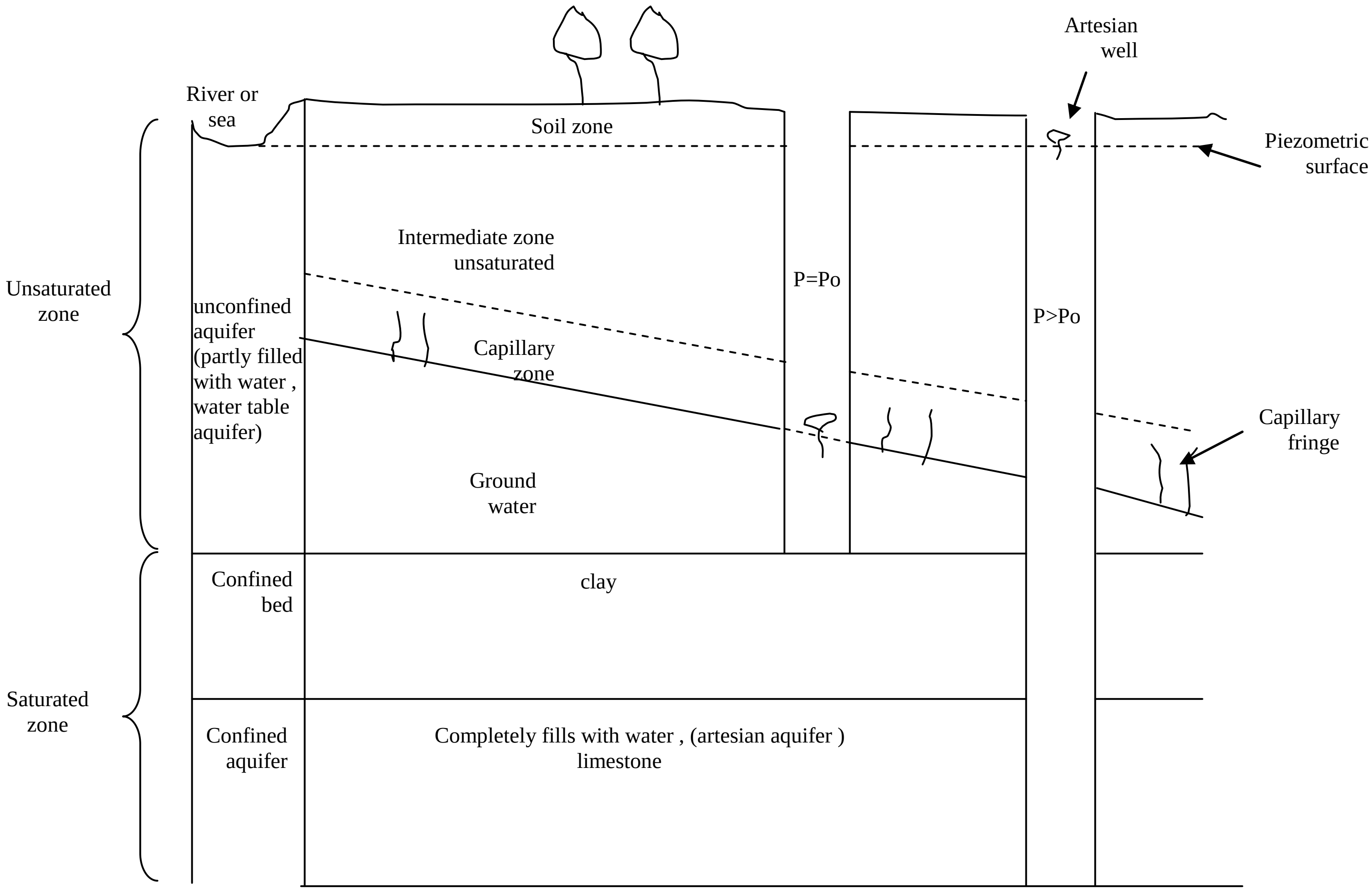
<u>Unconfined aquifer ;(water table ; shallow aquifer)</u>
water partly fill an aquifer the upper surface of the saturated zone is free to rise and decline

<u>Confined aquifer ;(artesian aquifer)</u>
Water completely fills an aquifer that is over line by a confined bed.

The wells is classified to :

Wells open to unconfined aquifers are referred to as water table wells , the water level in these wells indicates the position of the water table in the surrounding aquifer

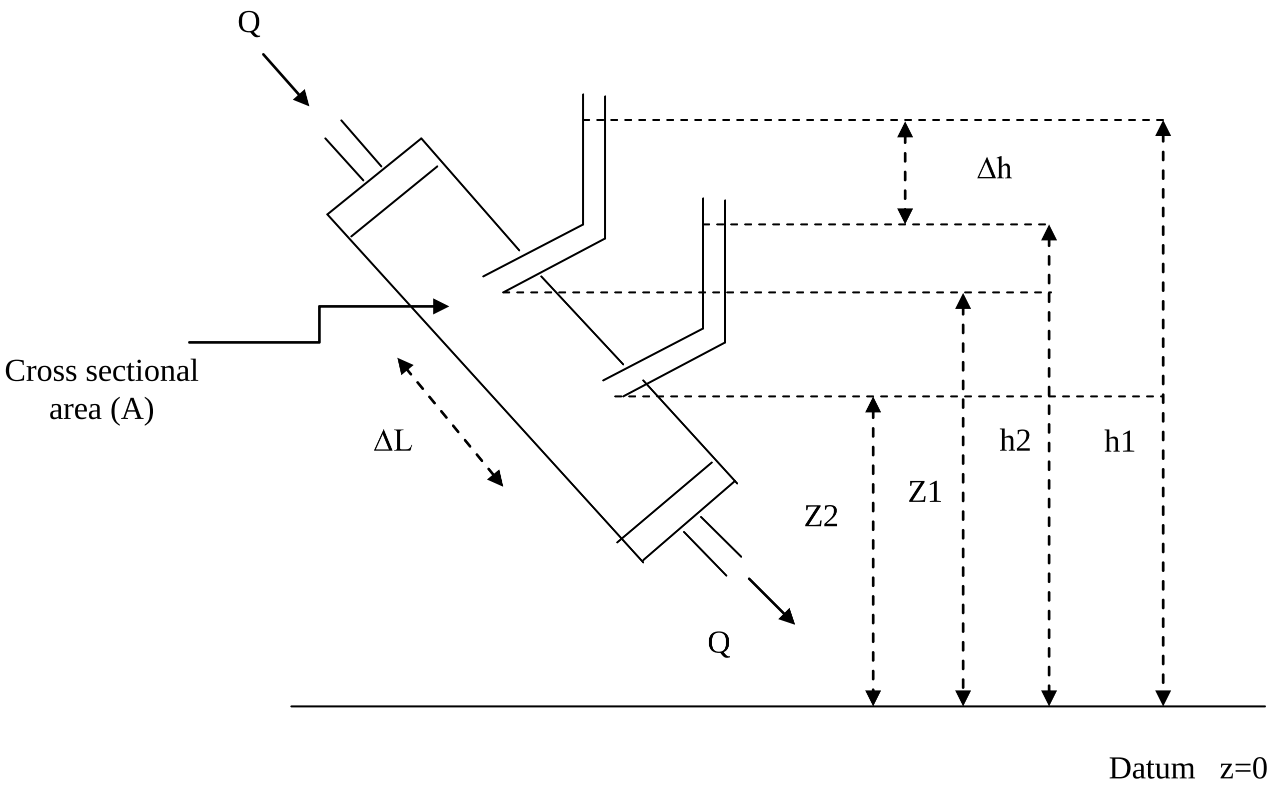
Well drilled into confined aquifers are referred to as artesian wells.
The water level in artesian wells stands at some height above the top of the aquifer but not necessary above the land surface
If the water level in an artesian well stands above the land surface the well is a flowing artesian well.
The water level in a well open to confined



Confined and unconfined aquifer

Darcy's Law

- Darcy described a laboratory experiment to analyze the flow of water through sands.
- The resulting of the experiment lead to the empirical law that now bears his name.
- The experiment was as below.



- A circular cylinder of cross section A is filled with sand stoppered at each end and out filled within flow and outflow tubes and a pair of manometers.
- Water is introduced into the cylinder and allowed to flow through it until such time as all the pores are filled with water and as get .

$$Q_{in} = Q_{out}$$

- Set an arbitrary datum at elevation Z=0
  - The elevation of the manometers intakes are Z1 and Z2 , and
  - The elevation of fluid levels are h1 and h2
  - ΔL = distance between manometer intakes

- The specific discharge through the cylinder (v) is defined as:

$$v = \frac{Q}{A}$$ (has dimension of velocity)

Darcy velocity , flux velocity

- The experiments carried out by Darcy showed that

$$v\alpha(h_1 - h_2)$$

$$v\alpha\frac{1}{\Delta L}$$

Darcy law $v = -k\frac{\Delta h}{\Delta l}$

- The negative sign indicate that the flow is in the direction of decreasing head

Or in differential form

$$v = -k\frac{dh}{dl}$$

Where h ; hydraulic head

$\frac{dh}{dl}$; hydraulic gradient

K; hydraulic conductivity which is a function of porous media and the fluid flowing through it (it is high for sand and gravel , low for clay and rock).

Since ; $v = \frac{Q}{A}$ then $\frac{Q}{A} = -k\frac{dh}{dl}$

$$Q = -k\frac{dh}{dl}A \quad = -kiA$$

i ; hydraulic gradient

- Darcy's law is valid for ground water flow in any direction in space.
- Seepage velocity $= = \frac{v}{n}$
- $n\alpha k$, high n give high k (but not for all the soil type).

Hydraulic conductivity and permeability

The hydraulic conductivity is a function of
- Porous media
- Fluid

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- Experiments have been carried out with ideal porous media consisting of uniform glass beads of diameter (d).
- When various fluids of density ρ and dynamic viscosity μ are run through the cylinder under constant $\frac{dh}{dl}$, the following relationships are observed

$v\alpha d^{2}$

$v\alpha\rho g$

$v\alpha\dfrac{1}{\mu}$

- Combining the above relation with Darcy law then

$v=-\dfrac{cd^{2}\rho g}{\mu}\dfrac{dh}{dl}$ ………………(1)

Where ; c= constant of proportionality (dimensionless constant)(depends on media properties) such as :

1. Distribution of grain sizes
2. The sphericity and roundness of the grain
3. The nature of packing

- Comparison with the original Darcy equation shows

$$K=\frac{cd^{2}\rho g}{\mu}$$

Where; ρ and μ are function of fluid alone

cd^{2} are function of medium alone

If we define $k=cd^{2}$ then $K=\dfrac{k\rho g}{\mu}$

Where k is a function of the medium and has dimensions (L^{2})

Hydraulic head and fluid potential

- The word potential refer to the transport due to gradient (from high to law)
- The fluid potential θ is to determined the potential gradient that controls the flow of water through porous media.
- Two obvious possibilities for potential quantity ; elevation and fluid pressure
- For Darcy experiment , flow occur down through the cylinder (from high to low elevation).
- If θ=90 (cylinder were placed in a horizontal position),gravity play no role.
- If θ=0 (vertical cylinder), flow would occur down through the cylinder from high elevation to low in response to gravity.
- Flow induced by → increasing the pressure at one end and decreasing it at the other

↓

That is lead to energy loss

- The classical definition of potential is the work done during the flow process.

- The work done in moving a unit mass of fluid between any two points in a flow system is a measure of the energy loss of a unit mass.

- Fluid through porous media is a mechanical process , the forces driving the fluid forward most overcome the frictional forces set up between the moving fluid and grains of the porous media.

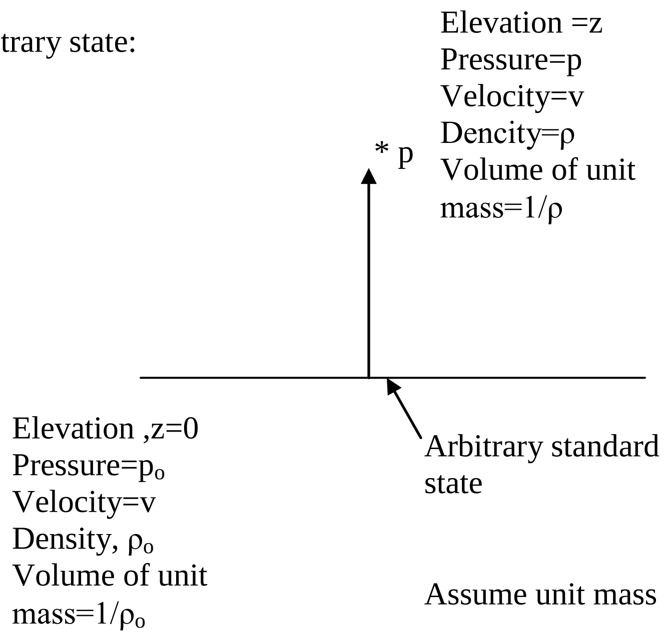
- The flow is therefore accompanied by an irreversible transformation of mechanical energy to thermal energy through the mechanism of friction resistance.

- Friction direction → from high mechanical energy /unit mass → lower

- Mechanical energy defined as work required to move a unit mass from point to point.

- Therefore ; fluid potential θ for low in porous media is the mechanical energy per unit mass of fluid.

- Consider an arbitrary state:



- The work required to lift a unit mass of fluid from the standard point to point p

- There are three components of work

1. Work required to lift the mass from elevation z=0 to z

$w_1 = mgz$ …………(1) represent the loss in potential energy

2. Work required to accelerate the fluid from velocity v=0 to v

$w_2 = \frac{mv^2}{2}$ ……..(2) represent loss due to kinetic energy

3. Work done on the fluid in raising the fluid pressure from p_o to p

$w_3 = m\int_{p_o}^{p} \frac{v}{m} dp = m\int_{p_o}^{p} \frac{dp}{\rho}$ ………..(3) represent loss due to elastic energy

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- The fluid potential θ (mechanical energy / unit mass) is the sum of $w_1 + w_2 + w_3$

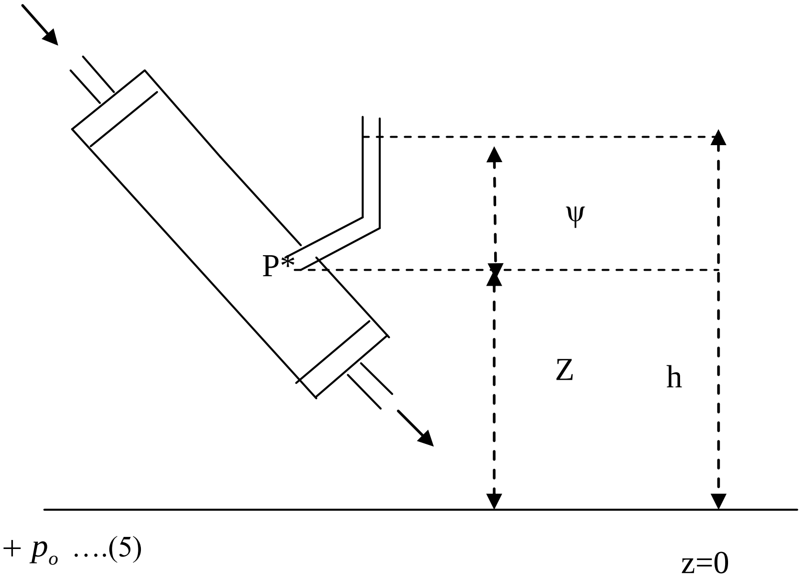
$\theta = mgz + \frac{mv^2}{2} + m\int_{p_o}^{p} \frac{dp}{\rho}$ for m=1

$\theta = gz + \frac{v^2}{2} + \int_{p_o}^{p} \frac{dp}{\rho}$ (Bernoulli equation)

- For porous media (v) are low , so the second term will ignored.
- For incompressible fluid (fluid will constant density so that ρ is not a function of p)

$\theta = gz + \frac{p - p_o}{\rho}$ …………(4)

- The relation to hydraulic head



At p the pressure is $p = \rho g \psi + p_o$ ….(5)

$$h = z + \psi$$

Elevation head pressure head

ψ = height of the liquid column above p

p_o = atmospheric pressure

$p = \rho g(h - z) + p_o$ …..(6) sub. (6) in (4)

$$\theta = gz + \frac{[\rho g(h - z) + p_o] - p_o}{\rho}$$

Then $\theta = gz + g(h - z) = gh$ ……(7)

Where: g is constant of the earth surface
Θ :units of energy / unit mass
h: units of energy / unit mass

- In ground water hydrology $p_o = 0$ and work in gage pressure (pressure above atmospheric pressure), therefore equation (6) will be
  $p = \rho g(h - z)$ ……..(8) sub. In (4)

  $$\theta = gz + \frac{p}{\rho} = gz + \frac{\rho g(h - z)}{\rho} = gh \quad …..(9)$$

  $gh = gz + \frac{p}{\rho}$ dividing by g

  $h = z + \frac{p}{\rho g}$.(10)

  The term of gage pressure yields
  $p = \rho g \psi$ ……(11) sub. In (10)

  $$h = z + \frac{\rho g \psi}{\rho g}$$
- If we put Bernoulli equation in term of head

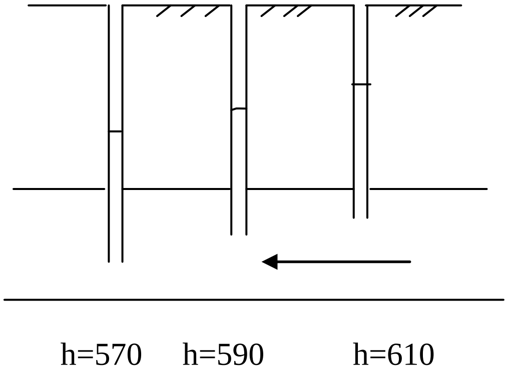
$h_t = h_z + h_p + h_v$
Where : total head= elevation head + pressure head+ velocity head

- Ground water moves in the direction of decreasing total head may or may not be in the direction of decreasing pressure head.

Piezometers and piezometers nets

Is a device for the measurement of hydraulic head at a “point” in the aquifer (it is tube or pipe).

- piezometer most be sealed along its length.
- It must be open to water flow at bottom and open to atmospheric at the top.
- The point of measurement is at the base not at the Level of the fluid surface.
- Piezometers are usually installed in groups so that this Can be used to determine the direction of ground water, which is called “piezometer nets”.



Heterogeneity and anisotropy of hydraulic conductivity

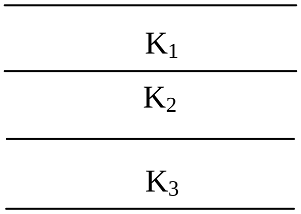
Homogeneity and Heterogeneity

If the hydraulic conductivity (k)m is independent of position within a geologic formation m the formation is homogenous.

If the (k) , is dependent on position within the geologic formation , the formation is heterogeneous

In heterogeneous formation k(x,y,z)=c

$K_1 \neq k_2 \neq k_3$



Isotropy and Anisotropy

If the hydraulic conductivity (k) is independent of the direction of measurement at a point in a geologic formation , the formation is isotropic.

If the hydraulic conductivity (k) varies with the direction of measurement at a point in a geologic formation , the formation is anisotropic.

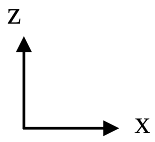
If an xyz coordinate system is set up , the (k) value will be

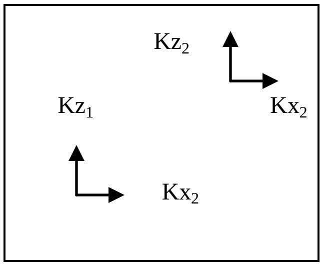
$K_x = k_y = k_z$ (isotropic formation)

$K_x \neq k_y \neq k_z$ (anisotropic formation)

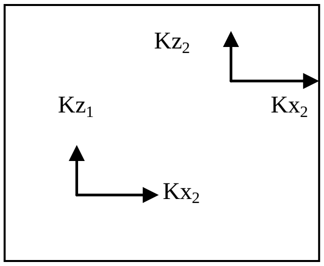
Figures below shows the four possible combinations of homogeneous , heterogeneity and isotropy , anisotropy.

The length of the arrow vectors is proportional to k_x and k_z values at the point (x_1,z_1) and (x_2,z_2)

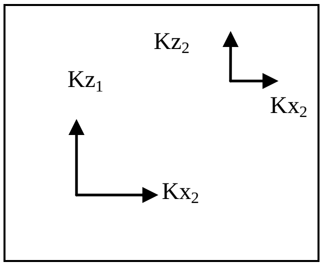




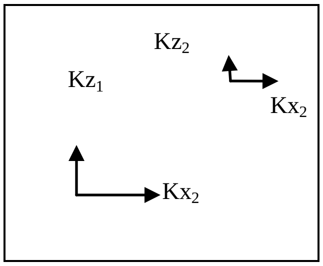
Homogeneous isotropic



Homogeneous anisotropic



Herogeneous isotropic



Hetrogeneous anisotropic

- For Homogeneous isotropic

$K_x(x,z) = k_z(x,z)=c$ for all (x,z) , c is constant

- For Homogeneous anisotropic

$K_x(x,z) = c_1$ for all (x,z)
$k_z(x,z)=c_2$ for all (x,z) , $c_1 \neq c_2$

Layered formation

There is a relation between layered heterogeneity and anisotropy each layer is homogeneoud and isotropic .

1. *Flow perpendicular to the layering*

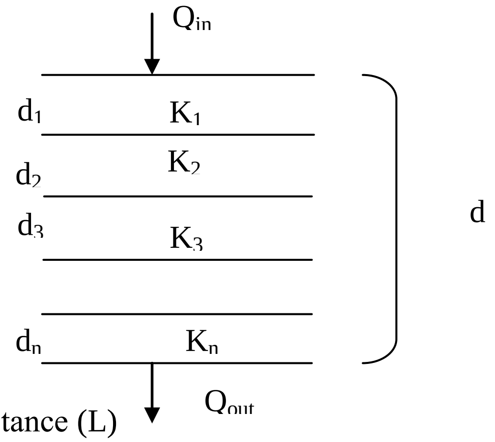
  - The specific discharge (v) must be the same entering the system as it is leaving

- Let Δh_1 be the head loss across the first layer and let Δh_2 be the head loss across the second layer ,and so on.
- The total loss $\Delta h = \Delta h_1 + \Delta h_2 + \ldots\ldots\ldots\ldots\ldots\ldots \Delta h_n$

So $v = \frac{k_1 \Delta h_1}{d_1} = \frac{k_2 \Delta h_2}{d_2} = \ldots\ldots\ldots\ldots = \frac{k_n \Delta h_n}{d_n} = \frac{k_z \Delta h_z}{d_z}$ (1)

Where k_z is equivalent vertical hydraulic conductivity for the system of layers.

- Solving equation (1)

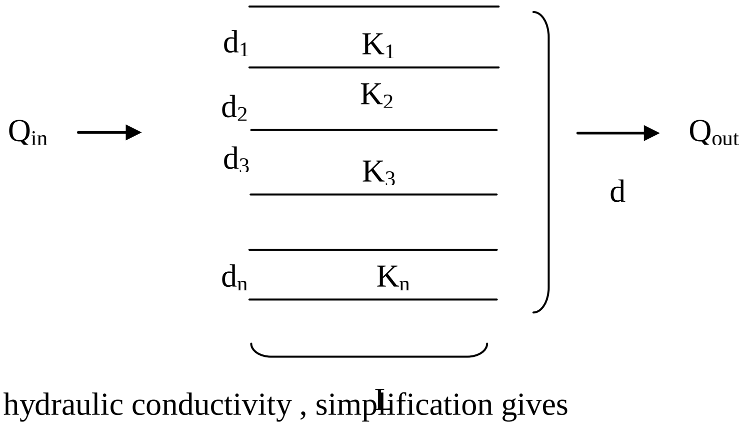


$$k_z = \frac{vd}{\Delta h} = \frac{vd}{\Delta h_1 + \Delta h_2 + \ldots + \Delta h_n}$$

$$= \frac{vd}{\frac{vd_1}{k_1} + \frac{vd_2}{k_2} + \ldots\ldots + \frac{vd_n}{k_n}} = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{k_i}}$$

2. *For flow parallel to the layering*

- Let Δh be the head loss over a horizontal distance (L)
- The discharge through a unit thickness of the system is the sum of the discharges through the layers.



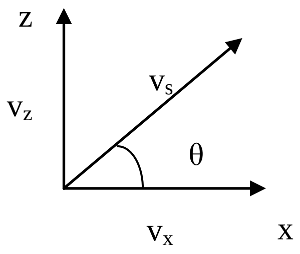
$$v = \frac{Q}{d}$$

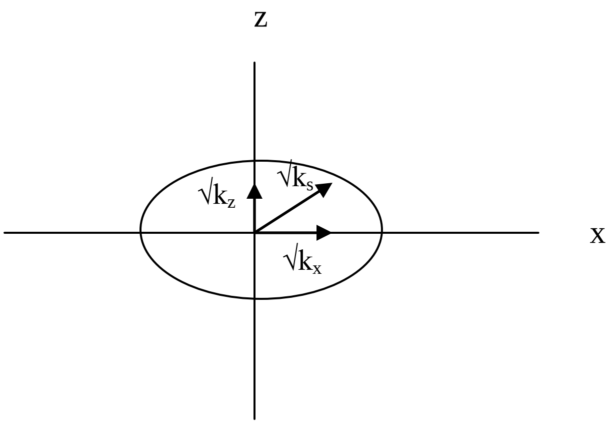
$$= \sum_{i=1}^{n} \frac{k_i d_i}{d} \cdot \frac{\Delta h}{L} = k_x \frac{\Delta h}{L}$$

Where k_x is an equivalent horizontal hydraulic conductivity , simplification gives

$$k_x = \sum_{i=1}^{n} \frac{k_i d_i}{d}$$

Hydraulic conductivity ellipsoid





Consider an arbitrary flow line in the xz plane in a homogeneous anisotropic medium with k_x and k_z .

- The flow line

$$v = -k_s \frac{\partial h}{\partial s} \qquad (1)$$

Where k_s is the unknown and it lies in range k_x __k_z

- v_s can be separate into its components v_x and v_z where

$$\begin{aligned} v_x &= -k_x \frac{\partial h}{\partial x} = v_s.\cos\theta \\ v_z &= -k_z \frac{\partial h}{\partial z} = v_s.\sin\theta \end{aligned} \qquad (2)$$

- Since h=h(x,z)

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x}.\frac{\partial x}{\partial s} + \frac{\partial h}{\partial z}.\frac{\partial z}{\partial s} \qquad (3)$$

- Geometrically , $\frac{\partial x}{\partial s} = \cos\theta$ and $\frac{\partial z}{\partial s} = \sin\theta$
- Substituting in equation (3) and combine with (1) and (2) yields

$$\frac{-v_s}{k_s} = \frac{-v_s.\cos\theta}{k_x}.\cos\theta + \frac{-v_s \sin\theta}{k_z}.\sin\theta$$

$$\frac{1}{k_s} = \frac{\cos^2\theta}{k_x} + \frac{\sin^2\theta}{k_z}$$

- In angular direction by setting $x = r\cos\theta$ and $z = r\sin\theta$ we get

$$\frac{r^2}{k_s} = \frac{x^2}{k_x} + \frac{z^2}{k_z}$$

<u>Darcy's law in three dimensions</u>

In the dimensional flow , v is a vector with components v_x, v_y, v_z

$$v_x = -k_x \frac{\partial h}{\partial x}$$

$$v_y = -k_y \frac{\partial h}{\partial y}$$

$$v_z = -k_z \frac{\partial h}{\partial z}$$

Since (h) is a function of x , y ,z the derivative must be partial , so the more generalized set of equations could be written in the form.

$$v_x = -k_{xx}\frac{\partial h}{\partial x} - k_{xy}\frac{\partial h}{\partial y} - k_{xz}\frac{\partial h}{\partial z}$$

$$v_y = -k_{yx}\frac{\partial h}{\partial x} - k_{yy}\frac{\partial h}{\partial y} - k_{yz}\frac{\partial h}{\partial z}$$

$$v_z = -k_{xz}\frac{\partial h}{\partial x} - k_{zy}\frac{\partial h}{\partial y} - k_{zz}\frac{\partial h}{\partial z}$$

If we put components of k in matrix form , it is known as the hydraulic conductivity tenser.

$$\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

- For general case $k_{xy} = k_{xz} = k_{yx} = k_{yz} = k_{zx} = k_{zy} = 0$

Limitation of Darcy's equation

1. Laminar flow $0.1 \le Re \le 10$
2. Homogeneous and isotropic flow $k_x = k_y = k_z$
3. No capillary zone
4. Steady state flow $\frac{dv}{dt} = 0$
5. Hydraulic head is the only driving force
6. Incompressible fluid (ρ constant)
7. Full saturated zone.

Un saturated flow

Darcy's law and the concepts of hydraulic head and hydraulic conductivity have been developed for saturated porous media and it is clear that some soils are partially filled with water and the other pores are filled with air . The flow of water under such conditions is called unsaturated or partially saturated flow.

Moisture content

If the total unit volume v_T of a soil or rock is divided into the volume of the solid portion v_s , the volume of the water v_w and the volume of air v_a the volumetric moisture content θ is defined as

$\theta = \frac{v_w}{v_T}$ like porosity , n

$\theta = n$ for saturated

$\theta \prec n$ for unsaturated

Water table

Is defined as the boundary between saturated and unsaturated zone and the fluid pressure (p) of it in the pores of a porous medium is exactly atmospheric , $p_o = 0$ (in gage pressure) $\longrightarrow$ $\psi = 0$

Since $h = \psi + z$, the hydraulic head at any point on water table must be equal to the elevation z of the water table at that point.

Negative pressure head

So we have $\psi \succ 0$ in the saturated zone
$\psi = 0$ in water table
$\psi \prec 0$ in the unsaturated zone

This reflects the fact that water in the unsaturated zone is held in the soil pores under surface tension forces.

Regardless of the sign ψ , the hydraulic head (h) is still equal $\psi + z$

However , above the water table , when $\psi \prec 0$, piezometers are no longer a suitable instrument for measurement of h, so head (h) must be obtained indirectly from measurement of ψ determined with tensiometers.

- لانه يحصل هناك تحدب بين الدقائق بسبب وجود الهواء والماء بسبب الخاصية الشعرية والشد السطحي التي تؤثر على الضغط وتؤدي الى ان يكون الضغط اقل من الصفرز

Saturated , unsaturated zone

For saturated zone

1. It occurs below the water table.
2. The soil pores are filled with water and $\theta = n$
3. The fluid pressure P is greater than atmospheric so the pressure head (measured as gage pressure) is greater than zero.
4. The hydraulic head must be measured with a piezometers.
5. The hydraulic conductivity k is a constant , it is not a function of pressure head ψ

For unsaturated zone

1. It occurs above the water table and above the capillary fringes.
2. The soil pores are only partially filled with water, the moisture content θ is less than the porosity.
3. The fluid pressure id less than the atmospheric, the pressure head ψ is less than zero.
4. The hydraulic head must be measured with a tensiometer.
5. The hydraulic conductivity and the moisture content are both function of the pressure head.

Transmissivity and stortivity

There are six basic properties of fluid and porous media that must be known in order to describe the hydraulic aspect of saturated ground water flow

For water

- Density, ρ
- Viscosity, ψ
- Compressibility , β

For media

- Porosity n (or void ratio , e)
- Permeability , k
- Compressibility , α

All the other parameters that are used to describe the hydro geologic propertied of geologic formations can be derived from these six properties.

Specific storage , Ss

Ss , of a saturated aquifer is the volume of water that a unit volume of water that a unit volume of aquifer releases from storage under a unit decline in hydraulic head (L^{-1}).

$$Ss = \rho g(\alpha + n\beta)$$

Where α = aquifer compressibility (media)

β =water compressibility

Ss, the volume/ unit volume / unit of decline in head

For a confined aquifer

- Transmissivity or transmissibility (T) is define as

  $T = kb$

  Where b = aquifer thickness

- Storitivity or storage coefficient (S) is define as

  $S = Ss.b$

The stortivity of a saturated confined aquifer of thickness (b) is the volume of water that an aquifer releases from storage per unit surface area of aquifer per unit decline in the component of hydraulic head normal to the surface.

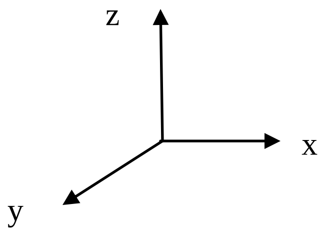
$$S = \rho g(\alpha + n\beta).b$$

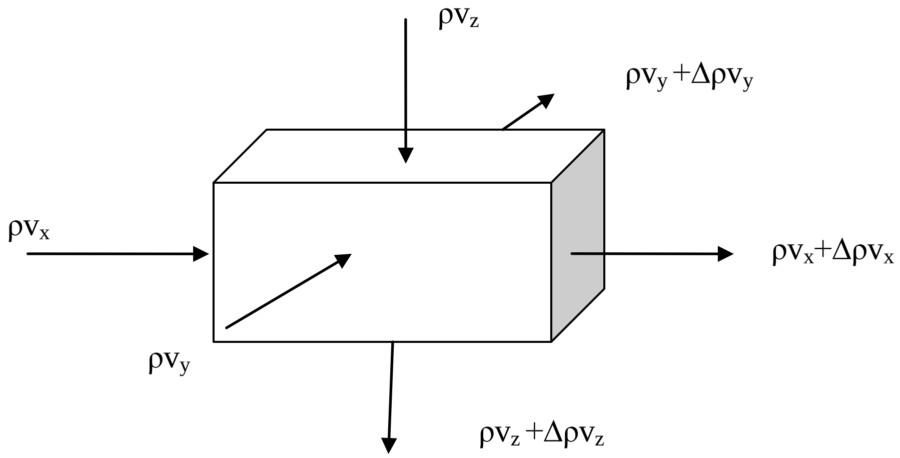
It is possible to define a parameter that couples the transmissivity properties T or k and the storage properties S , Ss which is the hydraulic diffusivity D

$$D = \frac{T}{S} = \frac{k.b}{Ss.b} = \frac{k}{Ss}$$

Equations of ground water flow

Consider a unit volume of porous media which is called an elemental control volume.





1. *Steady – state saturation flow*

The law of conservation of mass for steady – state flow through a saturated porous medium required.

Rate of fluid mass flow in = Rate of fluid mass flow out = 0

$$[\rho v_x - (\rho v_x + \Delta\rho v_x)]\Delta y.\Delta z + [\rho v_y - (\rho v_y + \Delta\rho v_y)]\Delta x.\Delta z + [\rho v_z - (\rho v_z + \Delta\rho v_z)]\Delta x.\Delta y =$$

Dividing by $\Delta x.\Delta y.\Delta z$ and take $\underset{\Delta x.\Delta y.\Delta z}{\lim \to 0}$

We know that $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

Then

$$-\frac{\partial}{\partial x}(\rho v_x) - \frac{\partial}{\partial y}(\rho v_y) - \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (1)$$

Conditions to simplify the equation

- For incompressible fluid $\rho(x.y.z) = \text{constant}$
- even if the fluid is compressible $\rho(x.y.z) \neq \text{constant}$.

  So the term b $\rho \frac{\partial v_x}{\partial x}$ are much greater than $v_x \frac{\partial \rho}{\partial x}$

  Then equation (1) simplifies to

  $$-\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z} = 0 \qquad (2)$$

Substitution of Darcy's law for v_x, v_y, v_z equation (2) yields

$$-\frac{\partial}{\partial x}\left(-k_x\frac{\partial h}{\partial x}\right)-\frac{\partial}{\partial y}\left(-k_y\frac{\partial h}{\partial y}\right)-\frac{\partial}{\partial z}\left(-k_z\frac{\partial h}{\partial z}\right)=0$$

For isotropic medium $k_x = k_y = k_z$ and if the medium is homogeneous then k(x,y,z)= constant , then we get

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

The partial differential equation is called "Laplace's equation"

The solution of the equation is a function h(x , y ,z) that describes the value of the hydraulic head at any point in the x , y , z flow fields

For anisotropic and homogeneous flow

$$k_x.\frac{\partial^2 h}{\partial x^2}+k_y.\frac{\partial^2 h}{\partial y^2}+k_z.\frac{\partial^2 h}{\partial z^2}=0$$

2. *Transient saturation flow (unsteady state)*

The law of conversation of mass for transient flow in a saturated porous medium required that the net rate of fluid mass flow into any elemental control volume be equal to the time rate of change of fluid mass storage within the element.

Rate of mass in – rate of mass out = the change of fluid mass storage within the element with Time

Then:

$$-\frac{\partial(\rho v_x)}{\partial x}-\frac{\partial(\rho v_y)}{\partial y}-\frac{\partial(\rho v_z)}{\partial z}=\frac{\partial(\rho n)}{\partial t}$$

$$-\frac{\partial(\rho v_x)}{\partial x}-\frac{\partial(\rho v_y)}{\partial y}-\frac{\partial(\rho v_z)}{\partial z}=\rho\frac{\partial n}{\partial t}+n\frac{\partial \rho}{\partial t}$$

Where:

$n\frac{\partial \rho}{\partial t}$ = the mass rate of water produced by an expansion of the water under a change in it's density , controlled by compressibility of the fluid, β

$\rho\frac{\partial n}{\partial t}$ = the mass rate of water produced by the compaction of the porous medium as reflected by the change in its porosity n, controlled by the compressibility of the aquifer, α

- The change in ρ and n are produced due to change in hydraulic head and we have
  $S=\rho g(\alpha+n\beta).b$

Then:

$$-\frac{\partial(\rho v_x)}{\partial x}-\frac{\partial(\rho v_y)}{\partial y}-\frac{\partial(\rho v_z)}{\partial z}=\rho.Ss\frac{\partial h}{\partial t}$$

Where :

$\rho.Ss\frac{\partial h}{\partial t}$ = time rate of change of fluid mass storage

The term $\rho\frac{\partial v_x}{\partial x} \succ\succ\succ v_x\frac{\partial \rho}{\partial x}$, that is leads to eliminate ρ from both sides

- Inserting Darcy equation

$-\frac{\partial}{\partial x}\left(-k_x\frac{\partial h}{\partial x}\right)-\frac{\partial}{\partial y}\left(-k_y\frac{\partial h}{\partial y}\right)-\frac{\partial}{\partial z}\left(-k_z\frac{\partial h}{\partial z}\right)=Ss\frac{\partial h}{\partial t}$ (this is the equation of flow for transient flow through a saturated anisotropic porous medium)

- If the medium is homogeneous and isotropic the equation become:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{Ss}{k}.\frac{\partial h}{\partial t}$$

Or

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{\rho g(\alpha+n\beta)}{k}.\frac{\partial h}{\partial t}$$

- The solution $h(x,y,z,t)$ describes the volume of the hydraulic head at any point in a flow field and at any time.
- To solve the above equation we need to know k,α,n for porous media, and ρ,β for fluid.
- For special case of horizontal confined aquifer of thickness $b, S=Ss.b$ and $T=kb$

For two dimensional form the above equation become

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S}{T}.\frac{\partial h}{\partial t}$$

- The solution required to know S,T
- This equation describe the change of h with respect to x,y at any time.

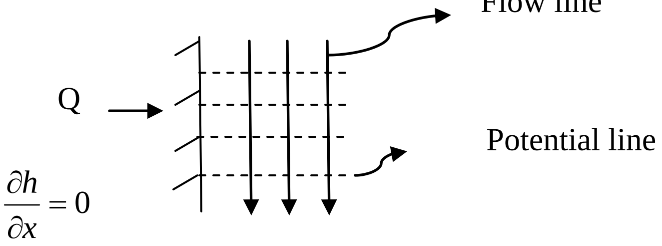
__The solution of continuity equation__

1. Graphical method
2. Analytical method
3. Numerical method(finite difference or finite boundary)

__Type of boundary condition in ground water__

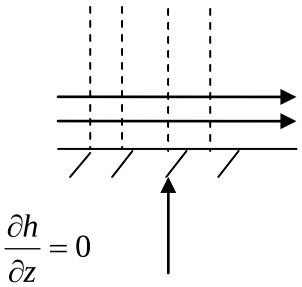
Three types of boundaries can exist for homogeneous isotropic and fully saturated region of flow with steady state condition.

1. Impermeable boundaries (Neuman boundaries)
   - No flow across the boundaries
   - The flow lines adjacent to the boundary must be parallel to it.
   - The equipotential lines must meet the boundary at right angles
   - $\frac{\partial h}{\partial z} = 0$ There is no flow across a flow line



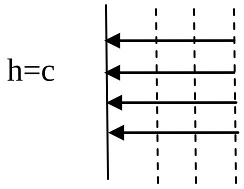
   - By involving Darcy's law and setting the specific discharge across the boundary equal to zero

$$v = -k\frac{\partial h}{\partial x} \Rightarrow 0 = -k\frac{\partial h}{\partial x} \Rightarrow \frac{\partial h}{\partial x} = 0$$

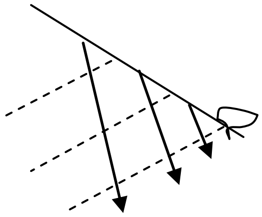


2. Constant head boundaries (Drichlet boundaries)

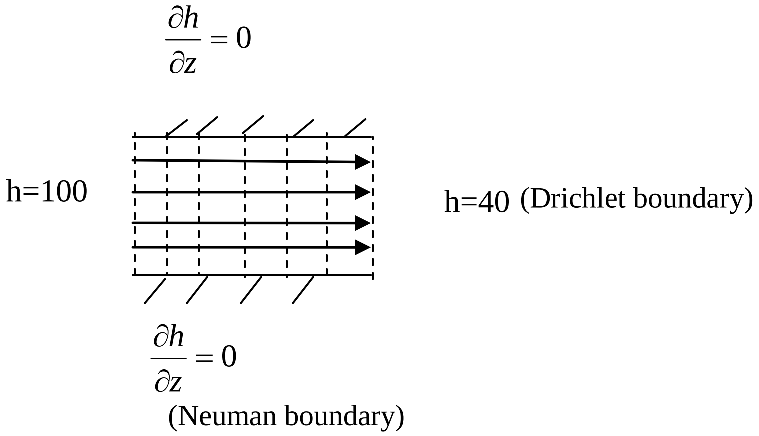
   - The boundary on which the hydraulic head is constant is an equipotential line.
   - The flow line must meet the boundary at right angles.
   - The equipotential line is parallel to the boundary
   - The mathematical condition , head =constant



3. Water table boundary
   - Pressure head $\psi = 0$ where $h = \psi + z = 0 + z \Rightarrow h = z$

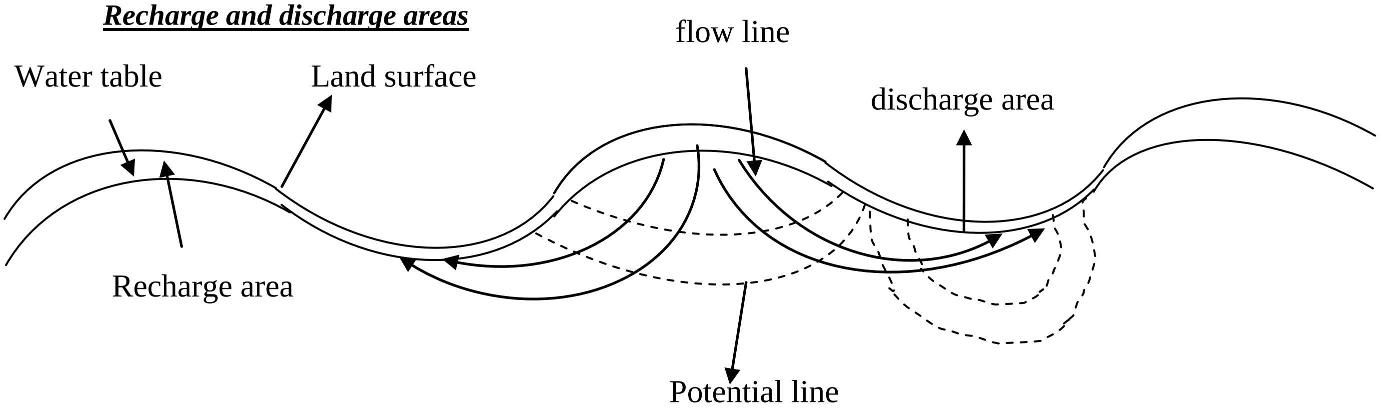


- For a recharge case the water table is neither a flow line nor an equipotential line . It is simply a line of variable but known h .



- The position of equal hydraulic head forms equipotential surface.

Recharge and discharge areas



- Ground water flow from high lands towards the valleys or recharge area
- *Recharge area*: the portion of the drainage basin in which the net saturation flow of ground water is directed away from the water table.
- *Discharge area*: the portion of the drainage basin in which the net saturation flow of ground water is directed toward from the water table.
- In the recharge area the water table , usually lies at some depth in the discharge area it is usually at or very near surface.

Boundary value problem

To fully define a transient boundary – value problems for subsurface flow , one need to know:

1. The size and shape of the region of flow.
2. The equation of flow within the region.
3. The boundary conditions around the boundaries of the region.

4. The initial condition of the region.
5. The spatial distribution of the hydraulic parameters that control the flow.
6. Mathematical method of solution.

- If the boundary – value problem is for a steady – state system requirement (4) is removed.

- Consider the simple ground water flow problem , the region ABCD contains a homogeneous , isotropic porous medium of hydraulic conductivity, k.

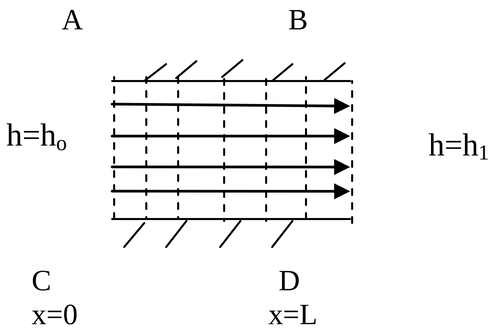
- The equation of flow for steady – state , saturated , homogeneous , isotropic media , the laplace equation

$$\frac{\partial^2 h}{\partial x^2} = 0$$

Using analytical solution for

B.C at x=0 h=h_o

X=L h=h_1



$$\frac{\partial^2 h}{\partial x^2} = 0$$

$$\frac{\partial h}{\partial x} = c_1 \Rightarrow \partial h = c_1 \partial x$$

$$h = c_1 x + c_2$$

Using B.C. (1) then

$$h_o = c_2 \Rightarrow h = c_1 x + h_o$$

Using B.C. (2) then

$$h1 = c_1 L + h_0$$

$$c_1 = \frac{h_1 - h_o}{L}$$

$$h = \frac{h_1 - h_o}{L} x + h_o$$

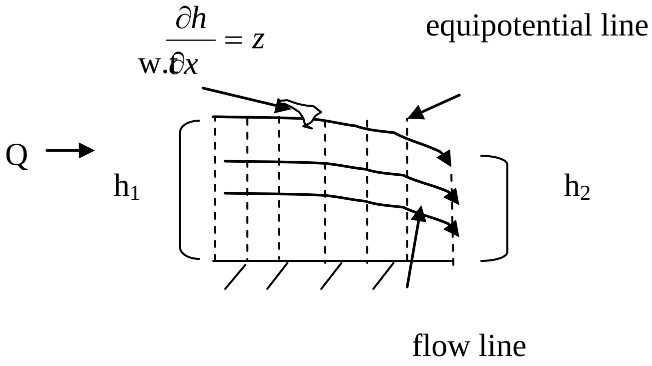
$$h = h_o - \left(\frac{h_o - h_1}{L} \right) x$$

Dupuit – Forchheimer theory of free surface flow

For flow in unconfined system bounded by a free surface an approach pioneered by Dupuit 1863 and advanced by Forchheimer 1930.

It is based on two assumptions

1. Flow lines are assumed to be horizontal and equipotential lines are vertical.
2. The hydraulic gradient is assumed to be equal to the slope of the free surface and to be invariant with depth.



- The theory is an empirical approximation to the actual flow field
- The theory neglect the vertical flow components
- In practice its value lies in reducing the two dimensional system to one dimension for the purpose of analysis.
- The discharge Q through a cross section of unit width perpendicular to the page

$$Q = -kA\frac{dh}{dx}$$

- Calculation based on the Dupiut assumptions favorable when:
    1. The slope of the free surface is small.
    2. The depth of the unconfined flow field is shallow

For unit area , $A = h(x) \times 1$

$$Q = -k.h(x)\frac{dh}{dx}$$

Analytical solution

$$\int_{x=0}^{x=L} Q\,dx = \int_{h=h_1}^{h=h_2} -k.h(x)dh$$

$$QL = k\frac{h_1^2 - h_2^2}{2}$$

$$Q = k\frac{h_1^2 - h_2^2}{2L}$$

- This equation is for unconfined flow
- The equation of flow for Dupiut – Forchheimer theory in a homogeneous isotropic medium can be developed from the continuity relationship

$$\frac{\partial Q}{\partial x} = 0$$

- This lead to $\frac{\partial^2 h^2}{\partial x^2} = 0$

▪ اذا كان unconfined فكل h تتحول الى h^2 لذلك نرجع الى الاشتقاق ونعوض بدل h تتحول الى h^2

$$\frac{\partial h^2}{\partial x} = c_1$$

$$h^2 = c_1 x + c_2$$

$$@\, x = 0 \Rightarrow h = h_1$$

$$c_2 = h_1^2$$

$$@\, x = L \Rightarrow h = h_2$$

$$h_2^2 = c_1 L + h_1^2$$

$$c_1 = \frac{h_2^2 - h_1^2}{L}$$

$$h^2 = \frac{h_2^2 - h_1^2}{L}x + h_1^2$$

$$h = \sqrt{h_1^2 - \frac{h_1^2 - h_2^2}{L}x}$$

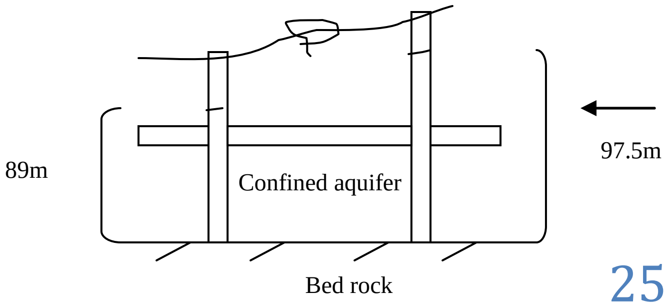
- h is the head at any distance in unconfined system

example : confined aquifer 33m thick and 7km wide has two observation wells 1.2km apparatus head reading at well (1) was 97.5m and at well (2) 98m , if k=1.2m/d what is :

1. the total daily flow of water through the aquifer.
2. The hydraulic head (h) at an intermediate distance (x) between the wells.

sol:

1.



$$Q = -kA\frac{dh}{dx}$$

$$A = 33 \times 7000 =$$

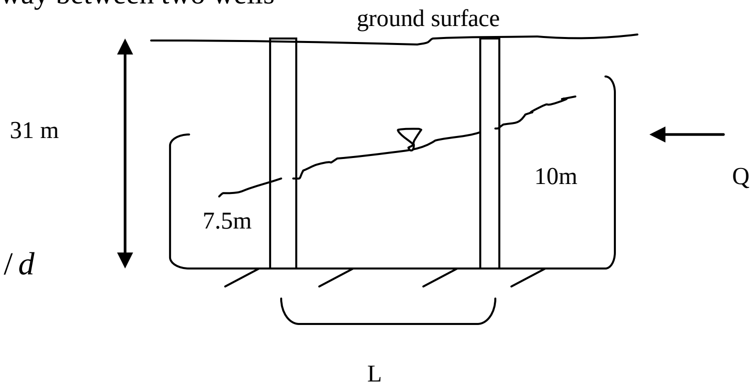
$$Q = -1.2(33 \times 7000).\left(\frac{97.5-89}{1200}\right) = 1963.5m^3/d$$

2. $$1963.5 = -1.2(33 \times 7000).\left(\frac{97.5-h}{600}\right)$$

$$h = 93.25m$$

Example: A water table sand aquifer with k=0.002 cm/s and n=0.27 the thickness 31m .at well (1) water level equal 21m below ground surface and at well (2) water level equal 23.3m below the ground surface , the distance between wells is 175 m find:

1. The discharge per unit width
2. Seepage velocity at well (1)
3. Water table elevation midway between two wells



Solution: for unconfined aquifer

1.

$$h_1 = 31-21 = 10m$$

$$h_2 = 31-23.5 = 7.5m$$

$$k = 0.002cm/s = 1.728m/d$$

$$Q = k\frac{h_1^2 - h_2^2}{2L}$$

$$Q = 1.728\frac{\left(10^2 - 7.5^2\right)}{2 \times 175} = 0.21m^3/d$$

2. $$v = \frac{Q}{nA} = \frac{0.21}{0.27(10 \times 1)} = 0.08m/d$$

$$h = \sqrt{h_1^2 - (h_1^2 - h_2^2)\frac{x}{L}} = \sqrt{10^2 - (10^2 - 7.5^2)\frac{175/2}{175}} = 8.84m$$

3. *or*

$$Q = k\frac{h_1^2 - h_2^2}{2L} \Rightarrow 0.21 = 1.728\left(\frac{10^2 - h_2{}^2}{2\left(175/2\right)}\right) \Rightarrow h_2 = 8.87m$$

Drilling and Installation of wells and Piezometers

The selection depend on :

- The purpose of the well
- Hydrological environment
- The quantity of water required
- The depth and diameter required
- Economic factor

Wells classified due to the method of construction:

1. Wells may be dug by hands
2. Driven in the form of well points
3. Bored by an earth auger or drilled by a drilling rig

There are three main types of drilling equipments

- Cable tool

طريقة بطيئة يحفر بالارتفاع للاعلى والنزول الى الاسفل لمجموعة من الادوات والتي تكون معلقة بالحبل ومن هذه الادوات اداة الحفر

- Rotary

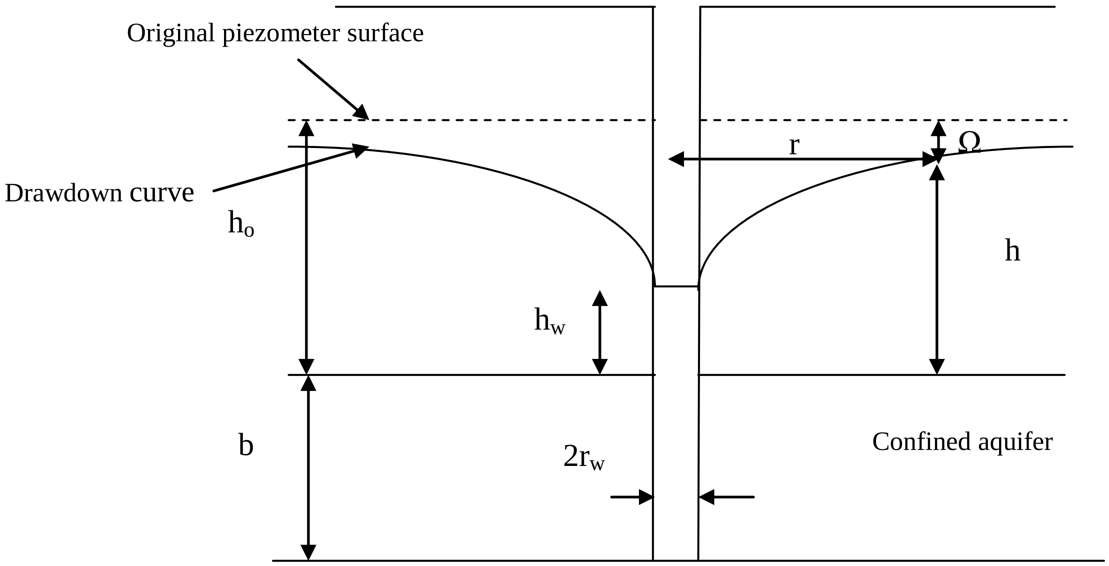
طريقة سريعة وقليلة الكلفة (يحفر ويدفع سائل الحفر ومن ثم تعاد السوائل الى السطح ، يستخدم سائل يدعى (drilling mud) مثل bentonitic clay in water

- Reverse rotary

Radial flow to a well

1. Steady flow in a confined aquifer

   Thiem's method



Using cylindrical coordinates, the well discharge Q at any radial distance r is given by :

$$Q = A.V = 2\pi rbk\frac{dh}{dr} \qquad (1)$$

Where A= area
V= velocity
b= thickness of the aquifer
$\frac{dh}{dr}$ = hydraulic gradient

- On integration and substitution of the boundary condition

$h=h_w$ when $r=r_w$ we get

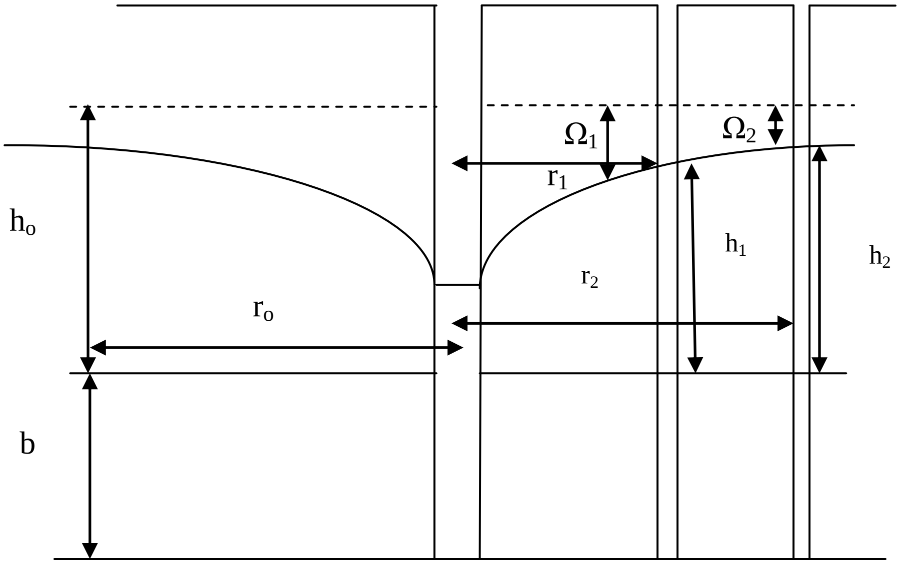
$$h - h_w = \frac{Q}{2\pi bk}\ln\frac{r}{r_w}$$

عندما نكامل معادلة Q

Where r_w = radius of the well
h_w = piezometric head at the well

or

$$Q = 2\pi bk\frac{h - h_w}{\ln\left(\frac{r}{r_w}\right)} \qquad (2)$$



- As an approximation a radius of influence r_o , can be assumed where h approaches h_o.
- On substituting these values in the above , the equilibrium or Thiem's equation for determination the well discharge is obtained.

$$Q = 2\pi bk \frac{h_o - h_w}{\ln\left(\frac{r_o}{r_w}\right)} \qquad (3)$$

or

$$Q = \frac{2\pi kb}{2.3} \cdot \frac{h_o - h_w}{\log \frac{r_o}{r_w}} \qquad (4)$$

- Equation (2) can also be used to determine the hydraulic conductivity of the aquifer
- Measurements of head h_1 and h_2 in two observation wells r_1 and r_2 respectively are sufficient to define the drawdown curve.

- K , is given by ; $k = \frac{Q}{2\pi b(h_2 - h_1)} \ln \frac{r_2}{r_1}$ (5)

- Since it is easier to measure the drawdown represented by Ω rather than piezometer head h is also written as

$$Q = \frac{2\pi kb(\Omega_1 - \Omega_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi kb(\Omega_1 - \Omega_2)}{2.3\log \frac{r_2}{r_1}} \qquad (6)$$

Example: the following drawdown were observed in a pump test on a well . The drilling indicated silty clay up to a depth of 20m , underline by a 25m thickness of a medium sand followed by fine sandy and clays sediment. The well screen was installed over a whole thickness of the aquifer of medium sand . A discharge of 150 m^3/hr was pumped for 15hr till the drawdown became steady determine the aquifer constants?

Observation well at	1.1m	30m	90m
Drawdown (m)	1.92	0.64	0.2

Solution:

$$Q = 150\frac{m^3}{hr} \times 24\frac{hr}{day} = 3600\frac{m^3}{day}$$

$$kb = \frac{Q.2.3\log\frac{r_2}{r_1}}{2\pi(\Omega_1 - \Omega_2)}$$

$$= \frac{3600 \times 2.3 \times \log\left(\frac{90}{30}\right)}{2 \times 3.14 \times (0.64 - 0.2)} = 1429\frac{m^3}{day}$$

- The same procedure can be fallowed using other combinations of piezometers . the results are given in the fallowing table.

r_1(m)	r_2(m)	Kb
1.1	30	1478.13
1.1	90	1465.55
30	90	1429
		Σ=1457.5

K=1457.5/25=58.3 m/d≈58m/d

Or

Alternatively the fallowing graphical procedure can be used:

1. The observed steady state drawdown of each observation well is plotted on single logarithmic (semi log) paper against it's distance r from the pumped well (r on log scale).
2. The best fitting straight line is drawn through the plotted points to give a distance drawdown carve.
3. The slope of this line ΔΩ is determined (i.s. the difference of drawdown per log cycle of r , giving $r_2/r_1=10$ or $\log r_2/r_1=1$

Then equation (5) will be

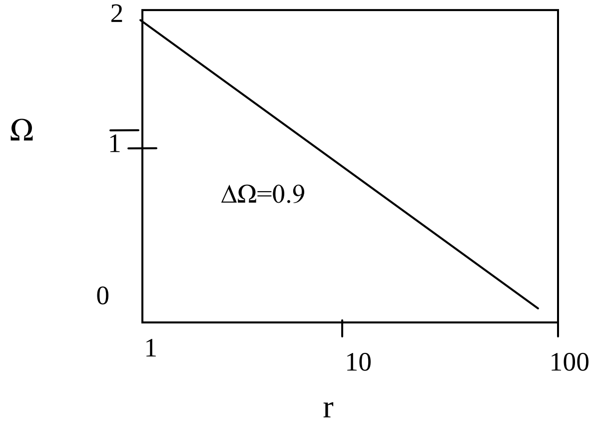
$$Q = \frac{2\pi kb\Delta\Omega}{2.3} \qquad (6)$$

4. The values of Q and ΔΩ are substituted in this eq. and the value of k or kb is calculated

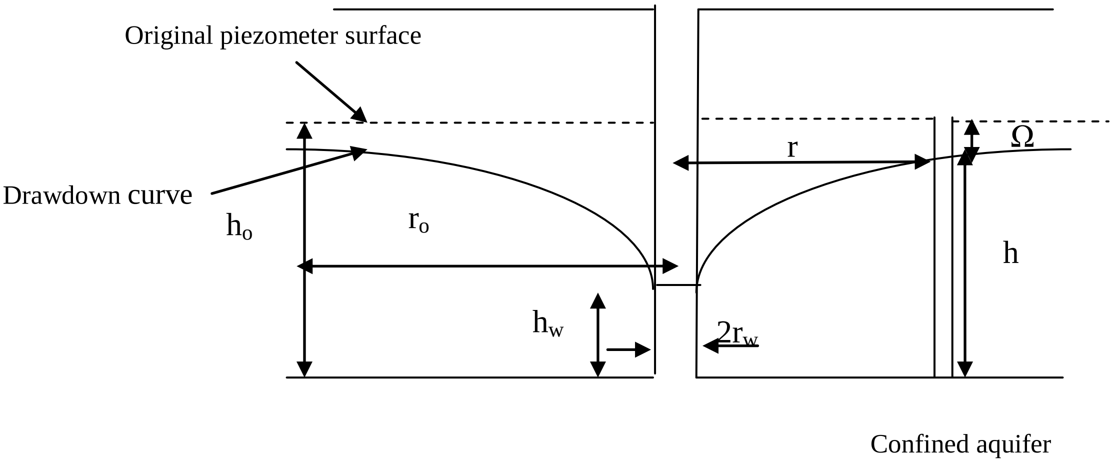
$$kb = \frac{2.3Q}{2\pi\Delta\Omega} = \frac{2.3 \times 3600}{2 \times 3.14 \times 0.9} = 1464m^2/d$$

$$k = \frac{1464}{25} = 58.5m/d$$

ناخذ اي نقطتين ونسقطهما على
محور Ω ونطبق الاتي
Slope = ΔΩ / logΔr



2. Steady flow in unconfined aquifer (Dupuit method)



Where h_o : is the original water table
Ω: actual drawdown

- The discharge in unconfined aquifer is:

$$Q = 2\pi r h k \frac{dh}{dr} \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

- On integrating and putting the limits of $h=h_w$ and at $r=r_w$

$$Q = \pi k \frac{h^2 - h_w^2}{\ln \frac{r}{r_w}}$$

- If the original water table is at h_o and the radius of influence is assumed to be r_o the discharge is given by:

$$Q = \pi k \frac{h_o^2 - h_w^2}{\ln \frac{r_o}{r_w}}$$

- The equation can be written in terms of drawdown since $h=h_o-\Omega$
- After integration eq.(7) between r_1 and r_2 ($r_2 > r_1$) gives

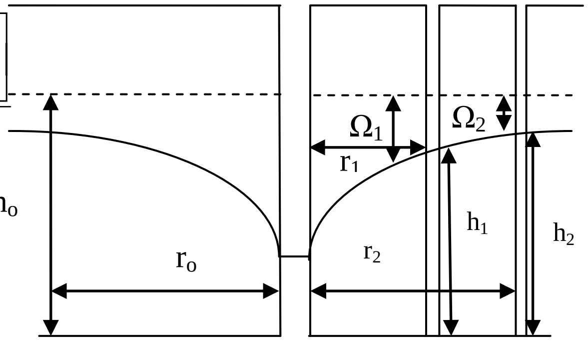
$$Q = \pi k \frac{h_2^{\;2} - h_1^{\;2}}{\ln \frac{r_2}{r_1}} = \pi k \frac{(h_o - \Omega_2)^2 - (h_o - \Omega_1)^2}{\ln \frac{r_2}{r_1}}$$

$$= 2\pi k \frac{\frac{h_o\left[\left(h_o^2 - 2h_o\Omega_2 + \Omega_2^2\right) - \left(h_o^2 - 2h_o\Omega_1 + \Omega_1^2\right)\right]}{2h_o}}{\ln \frac{r_2}{r_1}}$$ نضرب ونقسم البسط في 2ho/2ho

$$= 2\pi k ho \frac{\left[\frac{\left(2h_o\Omega_1 - \Omega_1^2\right)}{2ho} - \frac{\left(2h_o\Omega_2 - \Omega_2^2\right)}{2ho}\right]}{\ln \frac{r_2}{r_1}}$$

$$\overline{\Omega} = \frac{\left(2h_o\Omega - \Omega^2\right)}{2h_o}$$

let $$Q = \frac{2\pi k ho\left(\overline{\Omega_1} - \overline{\Omega_2}\right)}{\ln \frac{r_2}{r_1}}$$



- The equation is identical with Thiem eq.
- The same procedure can be fallowed to calculate the constants of the aquifer.

Measurement of Parameters

1. Laboratory Tests:

- Porosity: In principle the porosity (n) would be most easily measured by saturating sample measuring its volume v_t , weighing it and then oven drying it to a constant weight at 105C° . the weight of water removed could be converted to a volume knowing the density of water.
  This volume is equivalent to the volume of the void space v_v and we have

$$n = \frac{v_v}{v_t}$$

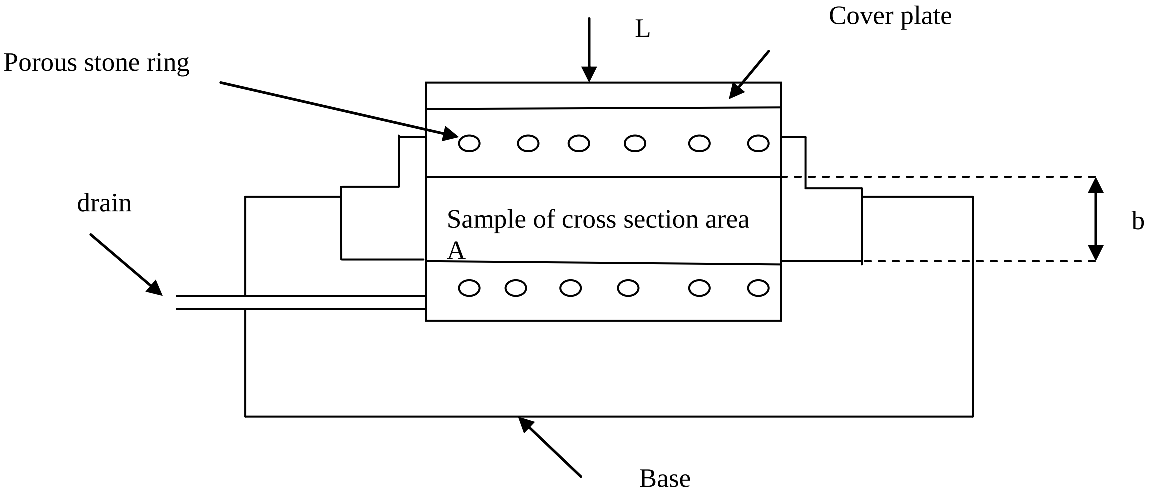
Because it is difficult to exactly and completely saturated a sample of given volume so it is more usual to use the relationship:

$$n = 1 - \frac{\rho_b}{\rho_s}$$

Where : ρ_b is the bulk mass density of the sample (equal to the oven – dried mass of the sample divided by its field volume)

ρ_s is the particle mass density (the oven dried mass divided by the volume of solid particle) and it is assumed for most mineral soils where great accuracy is not required equal to 2.65 g/ cm^3

- Compressibility : the compressibility of a porous medium is a measure of the relative volumetric reduction that will take place in a soil under an increased effective stress.
  Compressibility is measured in a consolidation apparatus
  In this test a soil sample is placed in a loading cell.

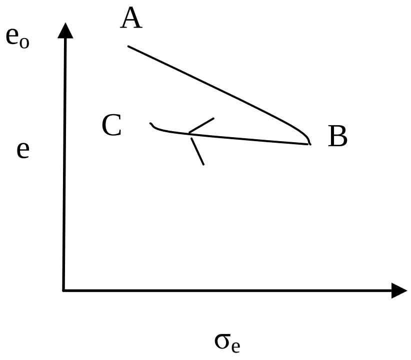


A load (L) is applied to the cell creating a stress σ , where $\sigma = \frac{L}{A}$

If the soil sample is saturated and the fluid pressure on the boundaries of the sample is atmospheric (the sample is free draining), the effective stress , which lead to consolidation of the sample is equal to the applied stress.

The reduction in the sample thickness (b) is measured after equilibrium is achieved at each of several loading increments.

The results are converted into a graph of void ratio (e) and effective stress σ_e



Compressibility (α) is determined from the slope of the graph

$$\alpha = \frac{-de(1+e_o)}{d\sigma_e}$$

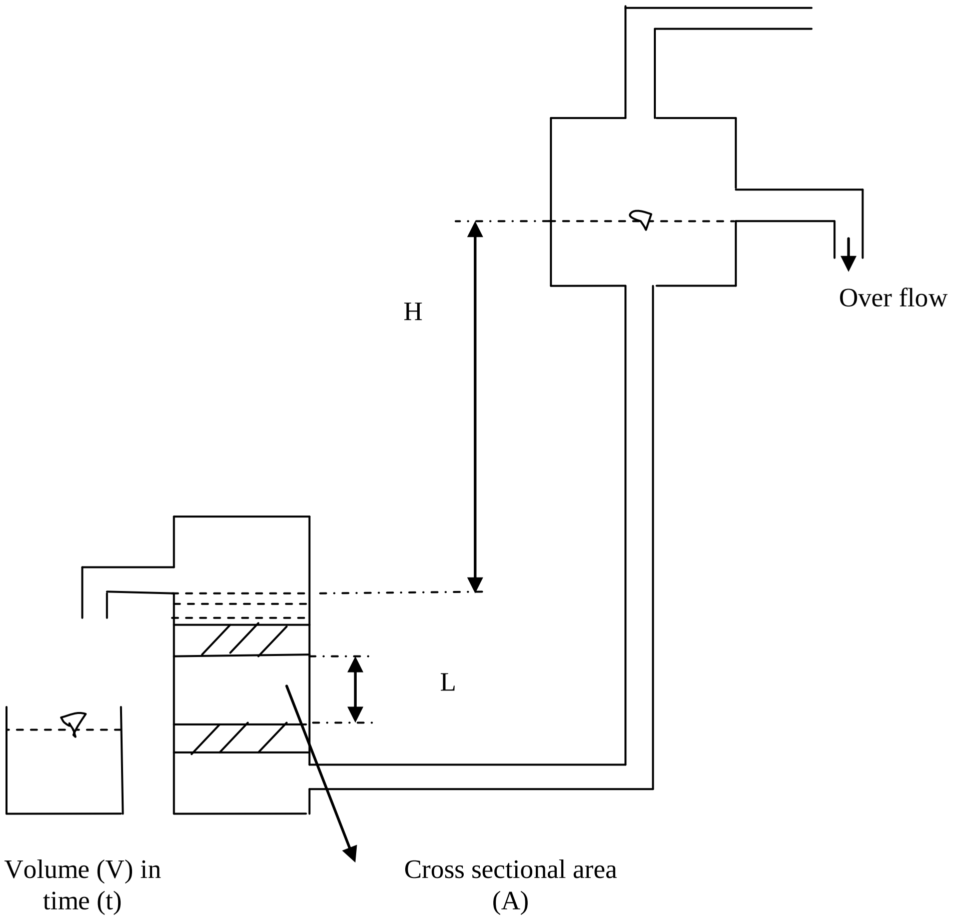
$$= \frac{-db/b}{d\sigma_e}$$

Where e_o is the initial void ratio

- Hydraulic conductivity: the saturated hydraulic conductivity of a soil sample can be measured with two types of laboratory apparatus

    a. Constant – head permeameters test

    In a constant – head test , a soil sample of length L and cross sectional area A is enclosed between two porous plates in a cylindrical tube and a constant – head differential H is set up a cross the sample.



$$Q = kA\frac{H}{L}$$

A simple application of Darcy's law lends to the expression

$$k = \frac{QL}{AH}$$

where : Q is the steady volumetric discharge through the system
it is important that no air become entrapped in the system

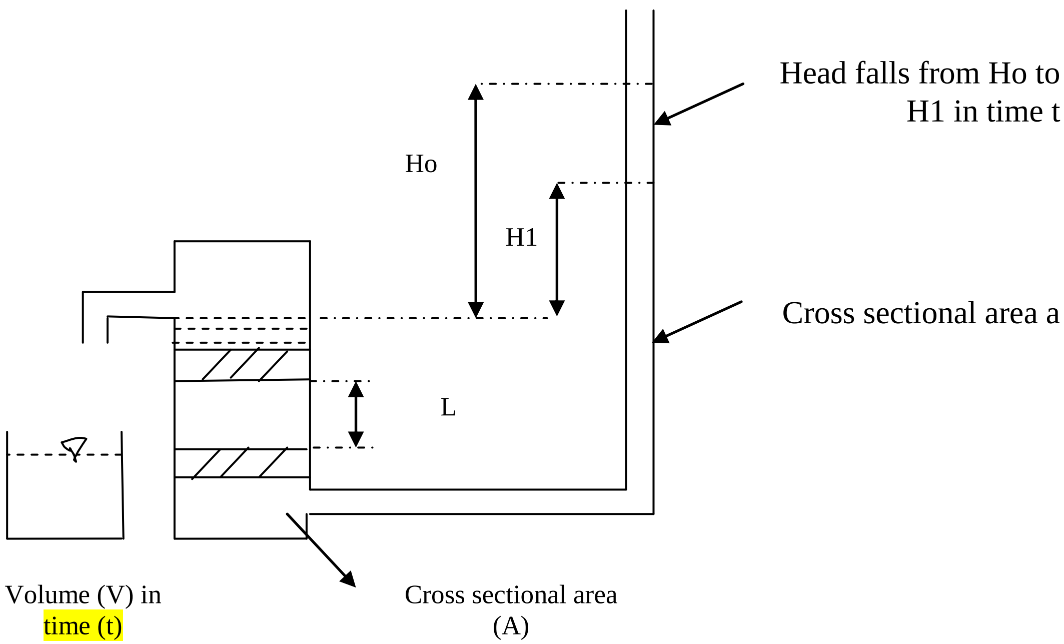
b. Falling – Head test

The head as measured in a tube of cross sectional area (a) , is allowed to fall from H_o to H_1 during time t

The hydraulic conductivity is calculated from

$$k = \frac{aL}{At}.\ln\left(\frac{H_o}{H_1}\right)$$

This equation derived from a simple boundary value problem that describes one – dimensional transient flow across the soil sample.
The lab test in not given the real value for k because it represent very narrow area around the sampling borehole.

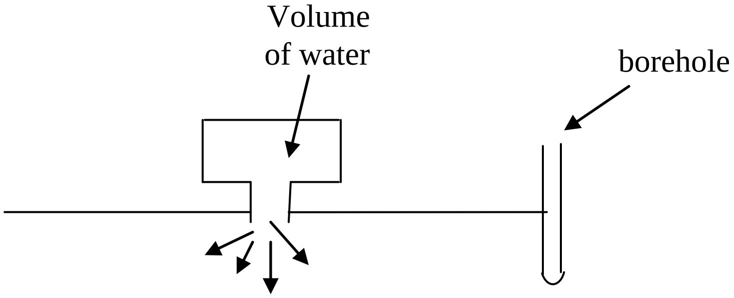


2. Measurement of parameters by piezometer test (field test)

   It is possible to determine is situe hydraulic conductivity values by mean of tests carried out in a single piezometers .
   Two tests are initiated by causing an instantaneous change in the water level in the piezometer through a sudden introduction or removal of a known volume of water , the recovery of the water level with time is then observed.

   - When water is removed , the test is called bail tests
   - When water is added , the test is called slug tests

   It is also possible to create the same effect by suddenly introducing or removing a solid cylinder of known volume.



The main limitation of this method it depend on the high – quality piezometer intake if the well point or screen is corroded or clogged measured values will be in accurate.

3. Measured of parameters by pumping tests

   This method is specifically suited to the determination of transmissivity (T) and Storativity (S) in confined and unconfined aquifer.

- Laboratory tests provide point values of the hydrological parameter and piezometer test provide in situe values representative of a small volume of porous media , pumping test provide in situe measurements that are averaged over a large aquifer volume

There are two graphical methods for calculating aquifer coefficients (T and S) from time – drawdown data:

1. The Theis method (on log – log plot curve)
2. The Jacob method (on semi- log plot curve)

▪ *The Theis method*

Theis (1935) utilized an analogy to heat – flow theory to arrive to an analytical solution to the equation of flow

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$ (In cylindrical coordinate)

And his solution, written in terms of the drawdown, is

$$h_o - h = \Omega = \frac{Q}{4\pi T} \cdot \int_{\frac{r^2 S}{4Tt}}^{\infty} \frac{e^{-u} du}{u} \qquad (1)$$

Where

h_o = original hydraulic head

h = hydraulic head at any radial distance ,r

Ω = drawdown , is the difference between h_o and h

Q = steady pumping rate

T = Transmisivity

S = Storativity

Where $u = \dfrac{r^2 S}{4Tt}$

Equation (1) can be written : $\Omega = \dfrac{Q}{4\pi T}.w(u)$

W(u) = the well function of u

The values of the integral w (u) can be tabulated for each of the several values of u as can be seen in table(1).

Example : a well is located in an aquifer with a conductivity of 15 m/d and S=0.005 . The aquifer is 20m thick and is pumped at a rate of 2725m^3/d , what is drawdown Ω at a distance 7m from the well after one day of pumping.

Sol:

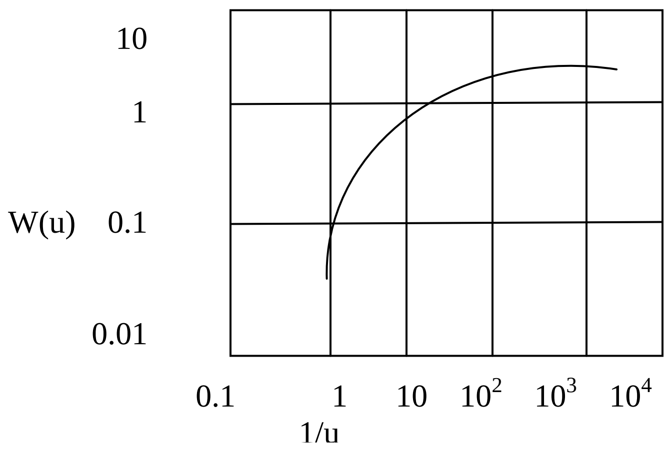
$$T = k.b = 15 \times 20 = 300 m^3 / d$$

$$u = \frac{r^2 .S}{4T.t} = \frac{7^2 \times 0.005}{4 \times 300 \times 1} = 0.0002$$

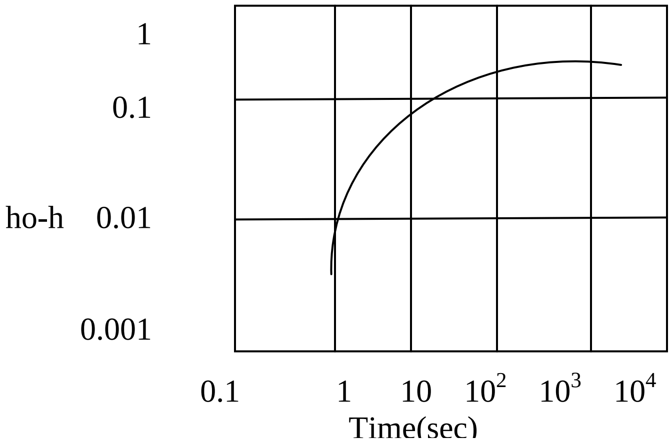
$$from\ table\ at\ u = 2 \times 10^{-4} \rightarrow w(u) = 7.94$$

$$\Omega = \frac{Q}{4\pi T}.w(u) = \frac{2725 \times 7.94}{4\pi \times 300} = 5.74m$$

Values of w(u) may be plotted against values of 1/u as can be seen in fig. 1



The values of Ω (ho-h) is plotted against time (t) at various value of r, on log-log paper



Both of the above two figures has the same form

In order to calculate T and S, Theis suggested the following graphical procedure

1. Plot the function w(u) versus (1/u) on log – log paper
2. Plot the measured time drawdown values Ω versus time t on log-log paper of the same size and scale as the w(u) versus (1/u) curve
3. Superimpose the field curve on the type curve keeping the coordinate axes parallel , adjust the curve until most of observed data points fall on the type curve.
4. Select an arbitrary match point and read off the paired values of w(u) , (1/u) , Ω , and t at the match point , calculate u and 1/u
5. Using these values together with the pumping rate Q and the radial distance r from well to piezometers , calculate T from the relationship

$$T = \frac{Q.w(u)}{4\pi(ho-h)} \quad or \quad T = \frac{AQw(u)}{(ho-h)}$$

6. Calculate S from the relationship $S = \frac{4uT.t}{r^2} \; or \; S = \frac{uT.t}{Br^2}$

- A and B are coefficients dependent on the units for the various parameters , for SI units with (ho-h) and r measured in meters , t in seconds Q in m^3/s and T in m^2/s
  A=0.08 and B=0.25

- For (o-h) and r measured in feet , t in days , Q in U. S. gal/min, T in U.S.gal/day/ft
  A=114.6 and B=1.87

Example : a well in a confined aquifer was pumped at a rate of 220gpm for about 8hr , the aquifer was 18ft thick , Thies Ω data for an observation well 824ft away are given below , find T , S and k

Time (min)	Ω (ft)	Time (min)	Ω(ft)
3	0.3	90	6.7
5	0.7	100	7
8	1.3	130	7.5
12	2.1	160	8.3
20	3.2	200	8.5
24	3.6	260	9.2
30	4.1	320	9.7
38	4.7	380	10.2
47	5.1	500	10.9
50	5.3		
60	5.7		
70	6.1		
80	6.3		

Solution :

Apply theis solution

1. Plot data on log-log paper
2. Overly on type curve (plot type curve) , 1/u , w(u)
3. Pick match point

U	1/u	W(u)
5	0.2	0.0011
2	0.5	0.049
1	1	0.219
0.5	2	0.56
0.1	10	1.82
0.01	100	4.04
And so on		

Then t and Ω

$$T = \frac{114.6 Q w(u)}{(ho - h)}$$

$$k = T/b$$

$$S = \frac{u.T.t}{1.87 r^2}$$

If the match point were taken at some point on the coincide portions of the curves , for quick calculations the match point will be taken anywhere on the overlapping field , for easy calculation take; w(u)=1 , 1/u=1

Then ho-h=2.4 ft, on curve =0.14m

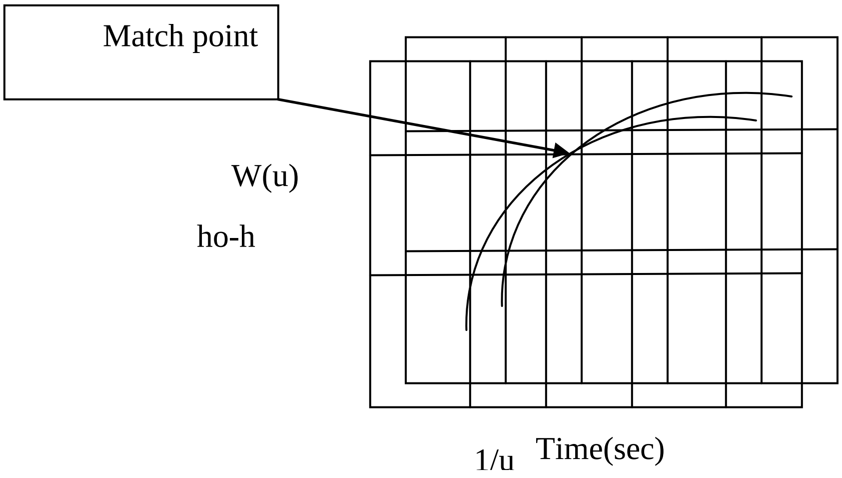
$$\frac{t}{r^2} = 6.06 \times 10^{-6}$$

$$T = \frac{114.6 \times Q \times w(u)}{ho - h} = \frac{114.6 \times 220 \times 1}{2.4} = 10500 gpd / ft$$

$$k = \frac{T}{b} = \frac{10500}{18} = 580 gpd / ft^2$$

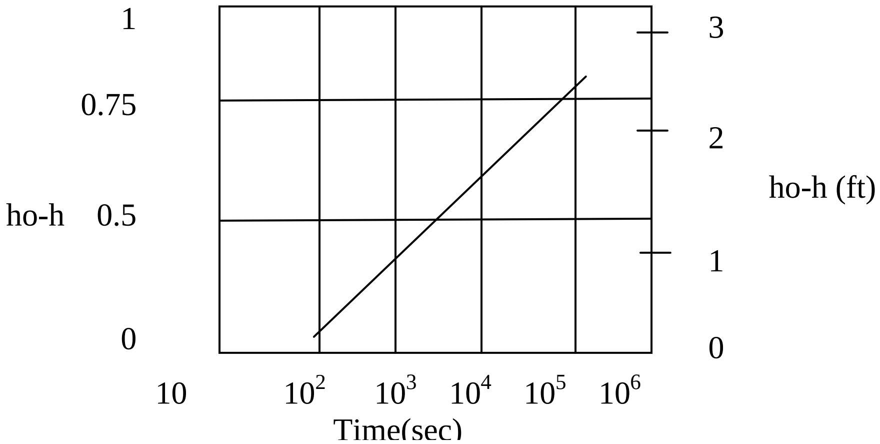
$$S = \frac{uT}{2693} \times \frac{t}{r^2} = \frac{1 \times 10500 \times 6.06 \times 10^{-6}}{2693} = 0.00002$$

Where 2693 is a converting factor



- ## The Jacob method

This method is approximation to Theis method , it is used when u are very small , the drawdown are plotted on semi log curve with time



T and S are calculated from the following equations:

$$T=\frac{2.3Q}{4\pi\Delta h}\ or\ \ T=\frac{CQ}{\Delta h}$$

$$S=\frac{2.25T.t_o}{r^2}\ or\ \ S=\frac{DT.t_o}{r^2}$$

Where t_o= is the intercept where drawdown line intercept the zero drawdown axis =440second

- C and D are coefficient that depend on the unit used
  For Δh and r in meters , Q in m^3/s and T in m^2/s , time in seconds then C= 0.18 D=2.25
  For Δh and r in ft , Q in U.S.gal/min and T in U.S. gal/d/ft , time in day then C=264 D=0.3

- By solving the previous example by Jacob method, plot data on semi log paper , find to = 5.2 min ,Δh=5.5 ft , then

$$T=\frac{CQ}{\Delta h}=\frac{264\times 220}{5.5}=10560 gal/d/ft$$

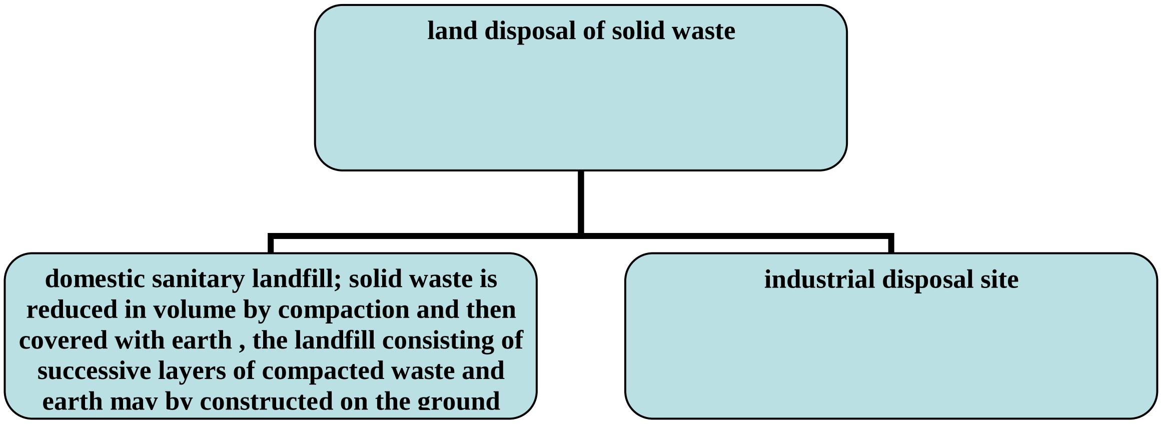
$$S=\frac{DT.t_o}{r^2}=\frac{0.3\times 10560\times\left(5.2\big/24\times 60\right)}{824^2}=0.000016$$

Ground Water Contamination

A ground water contamination is defined by most regulatory agencies as any physical, chemical, biological or radiological substance or matter in ground water.
The contaminations can be introduced in the ground water by naturally accruing activities, such as natural leaching of soil and other material with ground water.
Or ground water contamination can be introduced by human activities such as waste disposal, agricultural operations; human activities are the leading cause of ground water contamination.

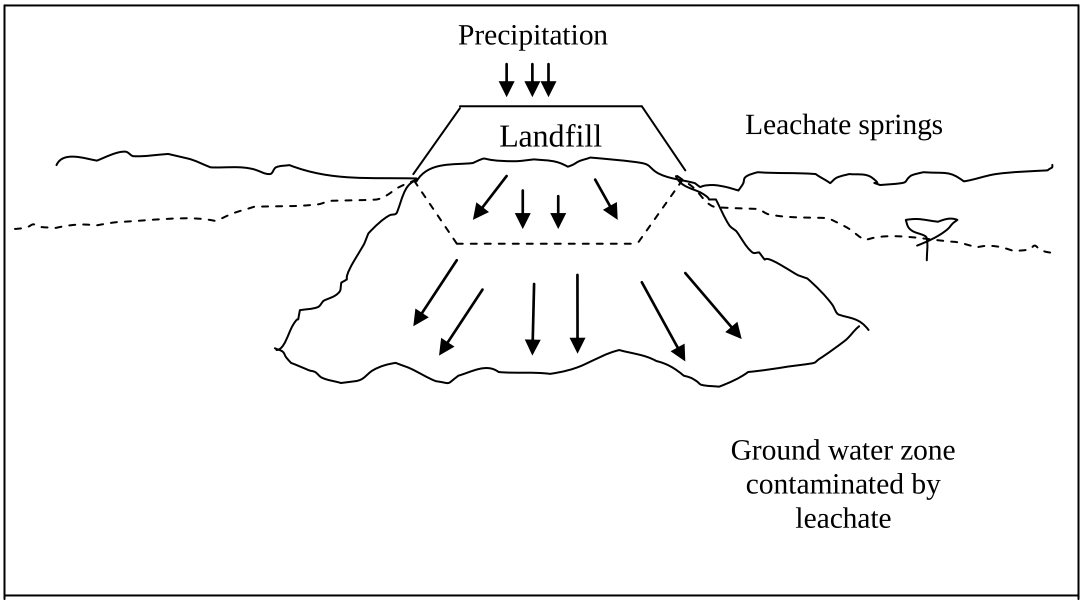
Sources of contamination

1. land disposal of solid waste

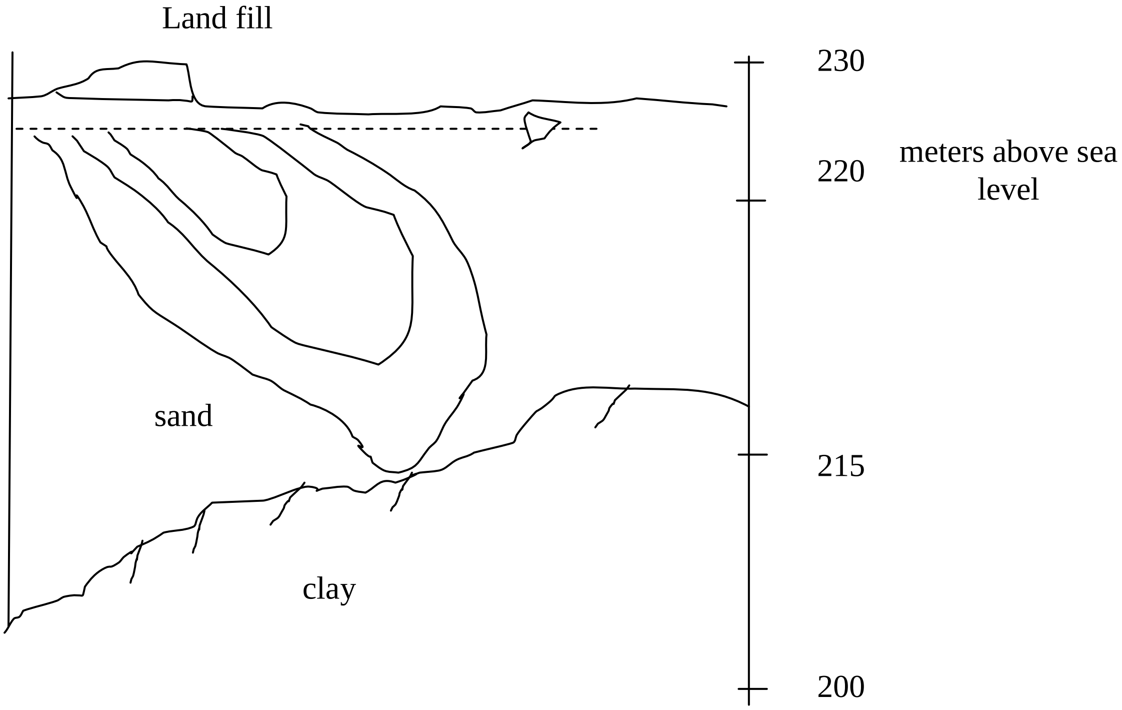


Leaching of dissolved solid contaminates to ground water by percolating water derived from rain or snowmelt (the liquid called leachate).

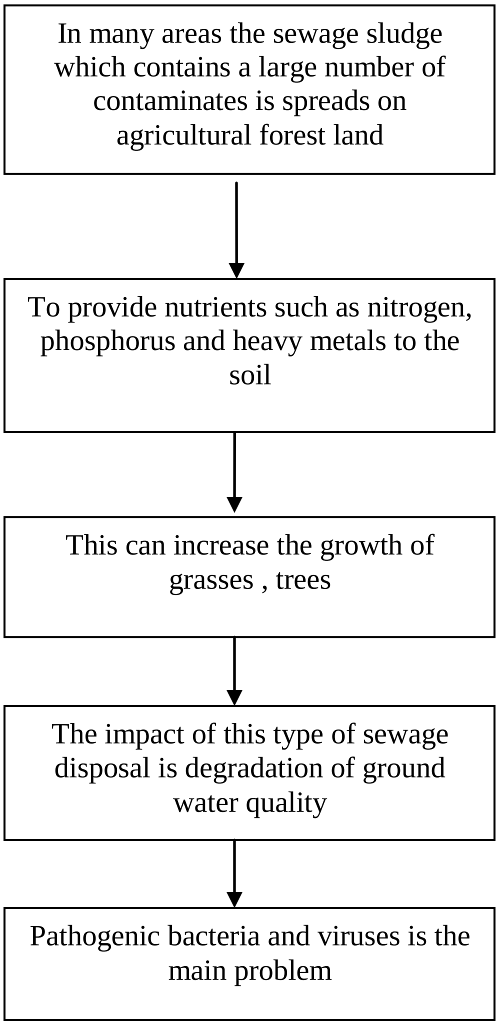
- Leachate from sanitary land fill contains large number of inorganic contaminates, organic contaminates and large amount of dissolved solid.
- In industrial disposal site leachate may contains toxic constituents from liquid industrial waste placed in landfill.
- Downward flow of leachate may threaten ground water outward flow causing leachate springs or seepage into streams or other surface water bodies.



- in situations where land fills are located in permeable material such as sand , gravel , leachate migration may cause contamination over area's many times larger than the areas occupied by the land fill



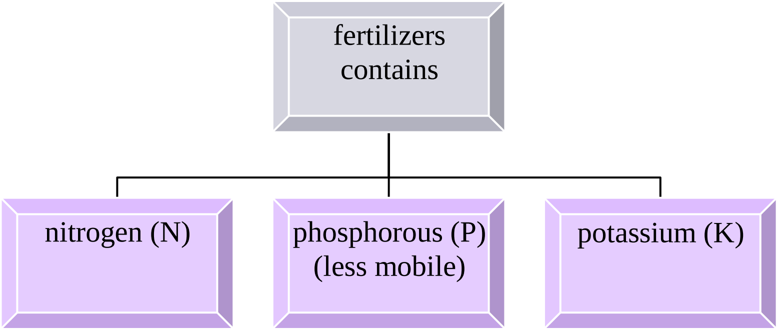
2. Sewage disposal on land
   Sewage is placed on or below the land surface in a variety of ways.



3. Agricultural activity

Among the main agricultural activities that can cause degradation of ground water quality are the usage of fertilizer and pesticides and the storage or disposal of animal wastes on land.

the most wide spread effects result from the use fertilizer (especially the type whom chemically manufactured).



- Nitrogen infiltrate in the form of NO_3 causing contamination of ground water.
- Although NO_3 is the main form , dissolved nitrogen also occurs in the form NH_4 , NH_3 , NO_2 , N_2 and N_2O.
- NO_3 is very mobile in ground water.

4. Petroleum leakage and spills

In industrialized countries thousands of steel gasoline storage tanks buried at filing stations and thousands of kilometers of underground pipelines carry petroleum products, etc.

The contamination of ground water by petroleum products from leaky tanks, pipelines or from spills.

5. Disposal of the radioactive waste

Nuclear fuel cycle: this expression refers to all the stages in the nuclear power industry in which nuclear fuel is developed and used in which radioactive waste is generated.

To avoid problems of subsurface radionuclide migration: the site must be located hydrogeologic environments that have long term containment capability.

To achieve capability: the site should have the following characteristics:

- geomorphic and structural stability
- isolation from fractured bedrock or other subsurface flow regimes
- Absence of subsurface flow lines that lead directly to the biosphere or to subsurface zones of potable water.

6. Deep well disposal
   Injection of liquid wastes mainly of industrial origin, the purpose of this procedure is to isolate hazardous substances from biosphere, but it causes ground water contamination.

Other sources

1. Large quantities of salts are applied to roads to combat adverse ice conditions during the winter months in Canada and USA, since salts such as $NaCl_2$ and $CaCl_2$ are highly soluble and relatively mobile in ground water.
2. Activity of mining industry are another cause of ground water contamination , the effect range from changes in ground water chemistry caused by mining to infiltration of leachate from tailing and another wastes.
3. Seepage from industrial waste lagoons is another cause of ground water contamination.

Source of ground water contamination: a contaminate is any dissolved solute or non aqueous liquid that inters ground water as a consequence of people's activities

Types of contaminates sources

a. Continuous: leakage from underground storage tank.
b. Discontinuous: Instantaneous or slug spill from quick spill or ruptured underground tank.

Other representation is:

1. point source
2. line source
3. area source

Type's contaminants

- reactive
- nonreactive
- dissolved
- non dissolved (immiscible in water oil and gasoline)

Transport Processes

The common starting point in the development of differential equation to describe the transport of solutes in porous materials is to consider the flux of solute into and out of a fixed elemental volume within the flow domain.

A conservative of mass

Net rate of change of mass of solute within the element=

Flux of solute into The element - flux of solute out of the element ± loss or gain of solute mass due to reactions

- After solving this equation

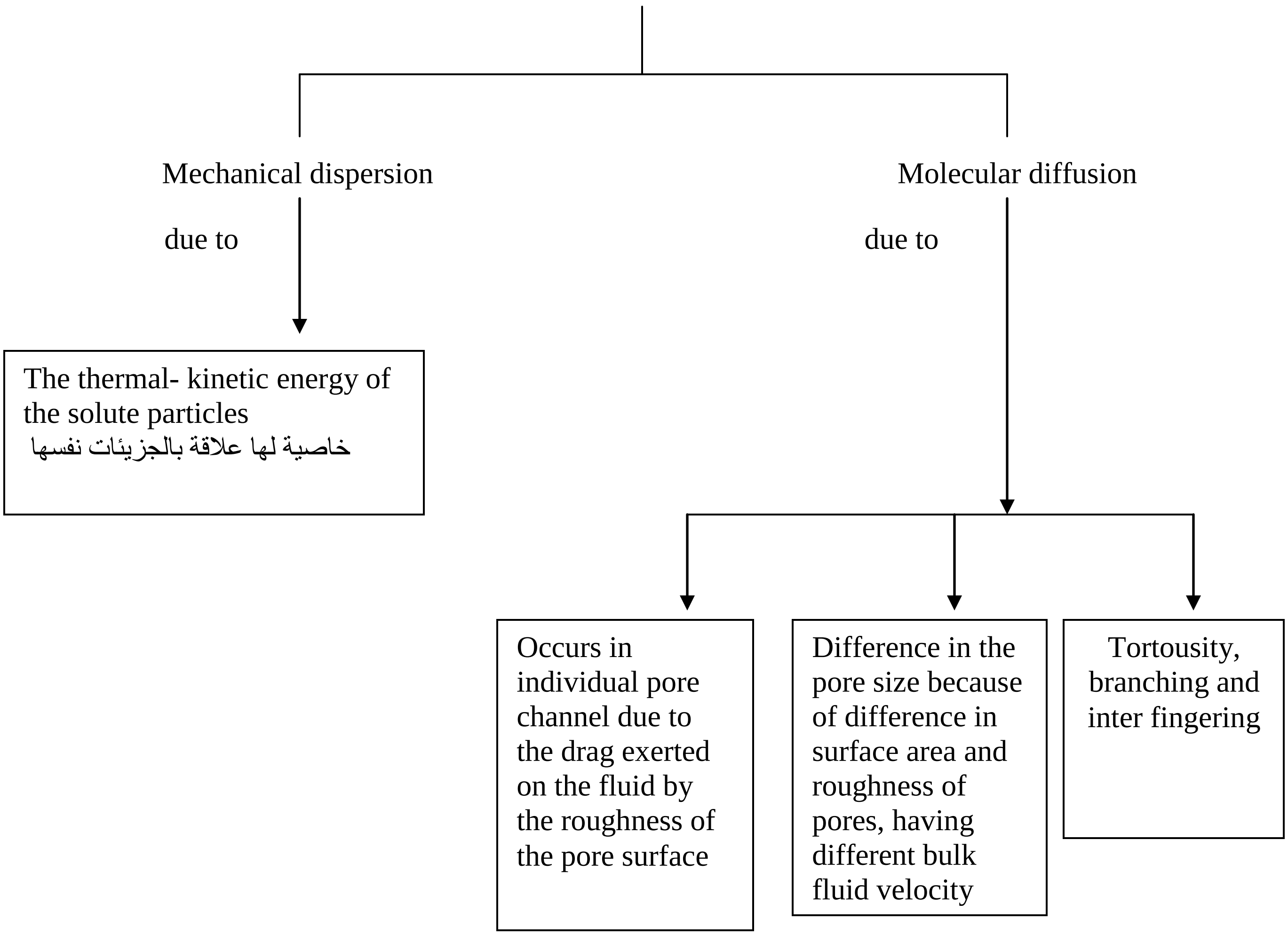
The physical processes that control the flux in and out of the elemental volume are advection and hydrodynamic dispersion.

Advection: is the component of solute movement attributed to transport by the flowing ground water and it is sometime called convection, or it may be defined as : solute are transported by the bulk motion of the flowing ground water.

The rate of transport = average liners ground water velocity (V)
$V = v/n$
where v is the specific discharge $=Q/A$
n=porosity

The hydrodynamic dispersion: there is a tendency for the solute to spread out from the path , that it would be expected to follow according to the advection hydraulics of the flow system .

Hydrodynamic dispersion occurs due to:



The thermal- kinetic energy of the solute particles
خاصية لها علاقة بالجزيئات نفسها

Occurs in individual pore channel due to the drag exerted on the fluid by the roughness of the pore surface

Difference in the pore size because of difference in surface area and roughness of pores, having different bulk fluid velocity

Tortousity, branching and inter fingering

Nonreactive constituents in Homogeneous media

The one – dimensional form of the advection – dispersion equation for nonreactive dissolved constituents in saturated, homogenous, isotropic materials under steady state, uniform flow.

$$\underbrace{D\frac{\partial^2 C}{\partial x^2}}_{\text{dispersion term}} - \underbrace{\bar{v}\frac{\partial C}{\partial x}}_{\text{Advection term}} = \frac{\partial C}{\partial t}$$

For three dimensions

$$\left[D_x\frac{\partial^2 C}{\partial x^2} + D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2}\right] - \left[\bar{v}_x\frac{\partial C}{\partial x} + \bar{v}_y\frac{\partial C}{\partial y} + \bar{v}_z\frac{\partial C}{\partial z}\right] = \frac{\partial C}{\partial t}$$

Where D= coefficient of hydrodynamic dispersion,
D= molecular diffusion + mechanical dispersion $=D_d+D_m$

$D_m = \alpha \bar{v}$

where α : characteristic property of the porous medium known as dispersivity (L)

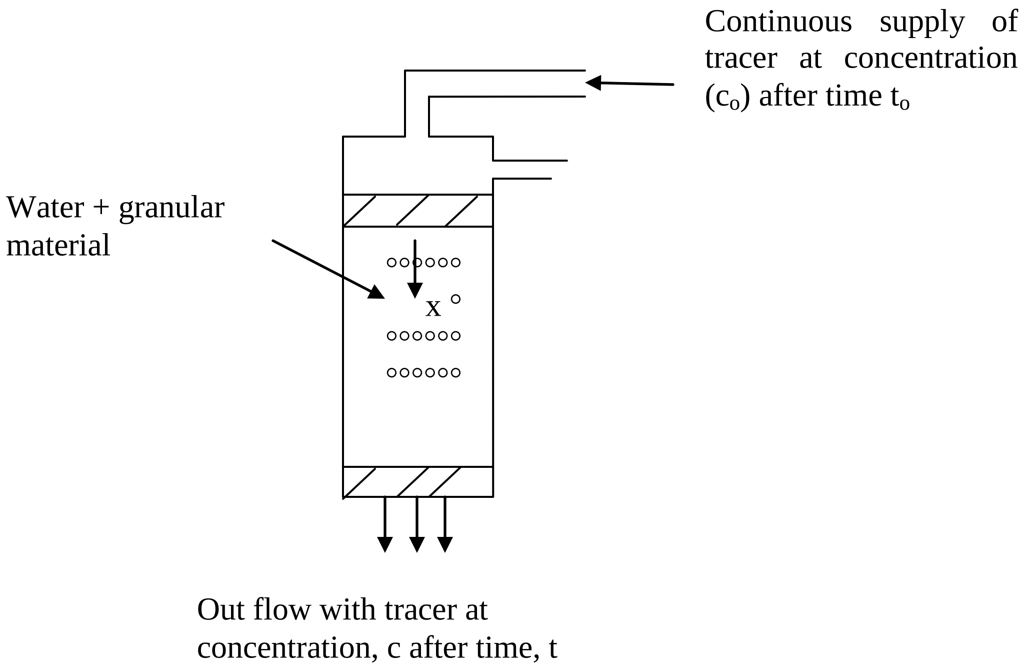
$\bar{v}$: ground water velocity

$D_d = D_o \tau$

where τ : tortuosity factor (0.6- 0.7)

D_o : free solute diffusion (from tables)

- the classical experiment that illustrate the physical meaning of the one dimensional form of advection – dispersion equation



- In this experiment a nonreactive tracers at concentration c_o, is continuously introduced into a steady – state flow regime at the upstream and of a column packed with a homogeneous granular medium.
- It is assumed that the tracer concentration in the column prior to the introduction is zero (Initial condition)
- It is convenient to express the tracer concentration in the column as a relative concentration defined as c/c_o

Where c : concentration in the column or in the outlet

  - the tracer input can be represented as shown in this fig.

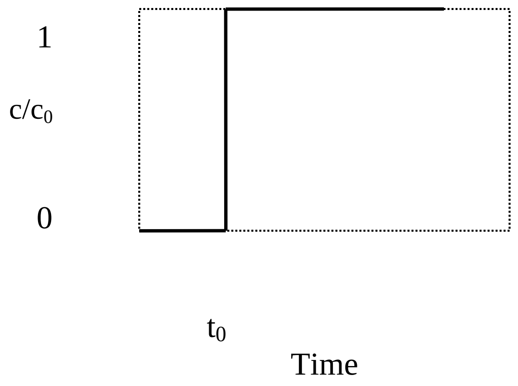


fig. (1)

- the concentration versus time relation of the column outflow known as breakthrough curve as shown in fig.(2)

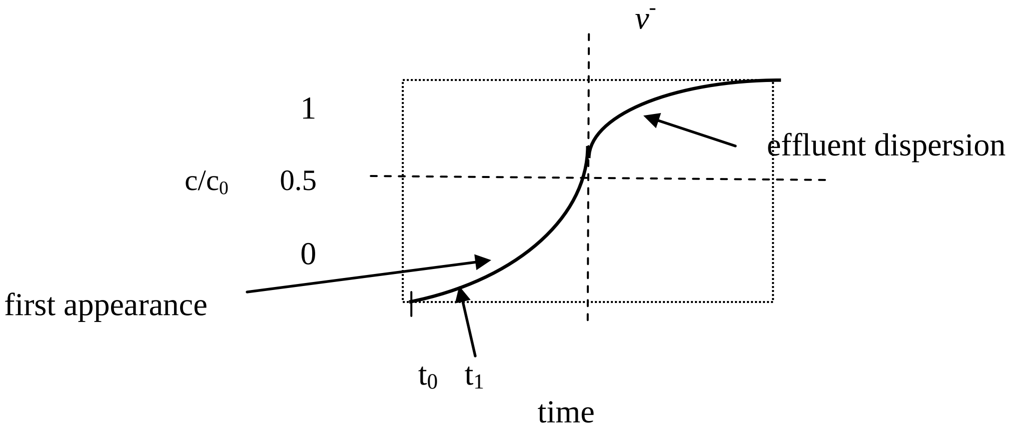
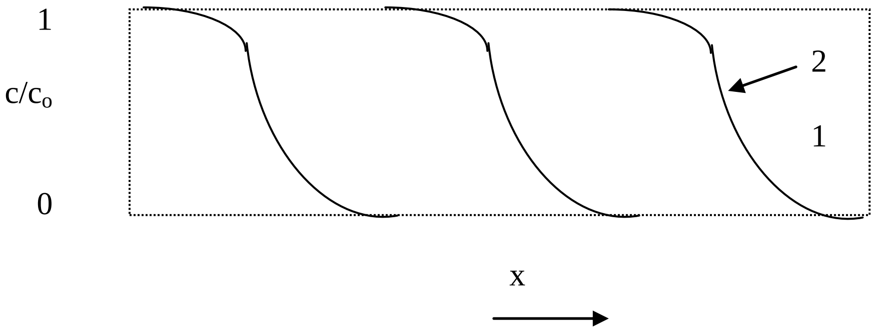


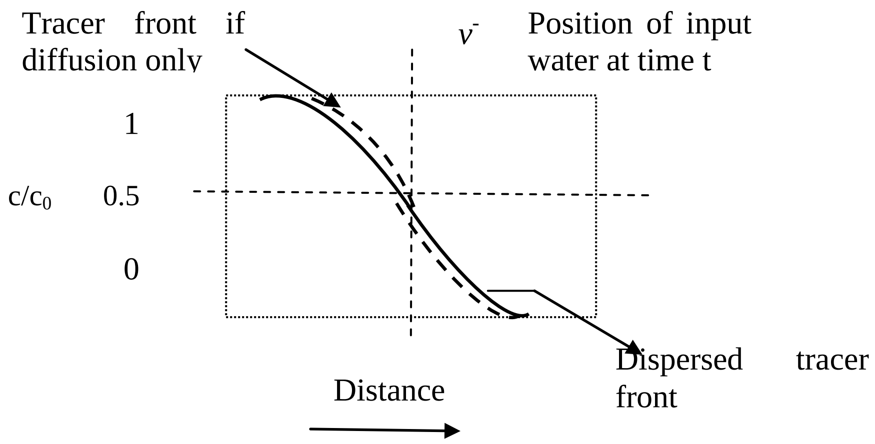
fig.(2)

step function من يدخل يتحرك plugومن ثم يخرج

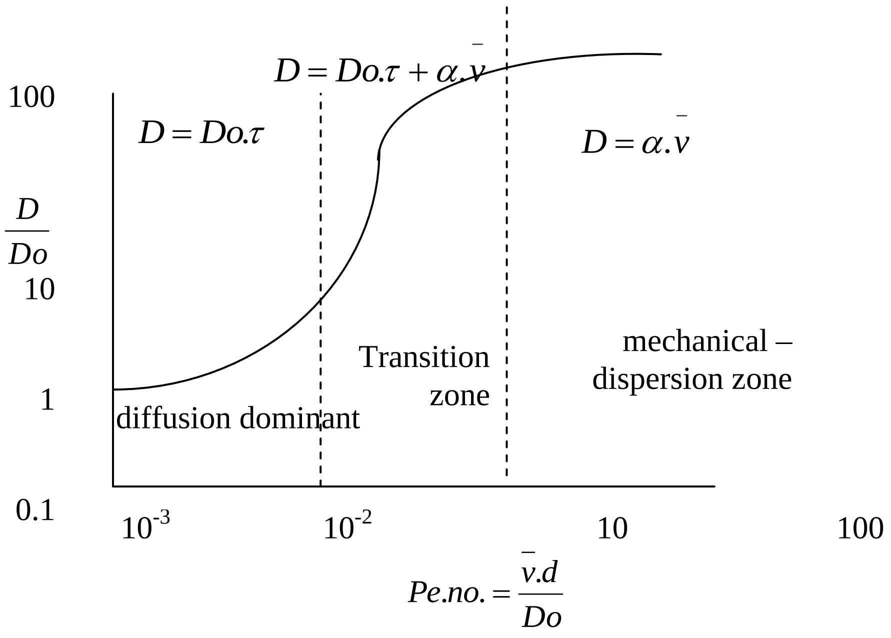
- It is assumed that the tracer moves through the column with no mechanical or macular diffusion
- In real situation , mechanical dispersion and molecular diffusion cause some of the tracer molecules to move faster than the average linear velocity of the water and some move slower.
- the figure below shows instantaneous "pictures" of the dispersion interface inside the column at times prior to breakthrough . The tracer front is spread out along the flow path.



- اندفاع الملوثات مستمر لمسافات متعددة من العمود
- The spread of the profile increases with travel distance, the positions represented by points 1, and 2 corresponding to times t_1 and t_2 in fig.2.
- The average linear velocity of the water in the column is determined by dividing the water input rate (Q) by nA.
  Where A is the cross sectional area of the column and
  n ;is the porosity.
- The spreading out of the concentration profile and breakthrough curve of traces through porous material is caused by both mechanical dispersion and molecular diffusion, the figure below shows a concentration profile for the experimental conditions represented before.



- The source material enters the column with a concentration c_o and displaced the original fluid , the advection front is located at the position $x = \bar{v}t$
- The concentration are at steady state and everywhere equal to the source concentration c_o.
- The contribution of molecular diffusion to the spread of the curves is indicated schematically
- At low velocity ; diffusion is the important contributor to the dispersion and therefore the coefficient of hydrodynamic dispersion equals to the diffusion coefficient. $D = D_d$
- At high velocity , mechanical mixing is the dominate dispersive process $D = D_m = \alpha\bar{v}$
- Large dispersivity of the medium produces greater mixing of the solute front as it advaces.
- Laboratory experiments on tracer migration in saturated homogeneous granular materials relations between the influence of the diffusion and mechanical dispersion.



- The dimensionless parameter $\frac{\bar{v}.d}{Do}$ is known as the Peclet number.

The exact shape of the relation between Pe. no. and D/D_o depends on the nature of the porous medium and on the fluid used

$$D\frac{\partial^2 C}{\partial x^2} - \bar{v}\frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$

Solved by Ogata (1970) taking the fallowing initial boundary condition :

$c(x,0) = 0 \quad @ \quad x \geq 0$

$c(0,t) = c_o \quad @ \quad t \geq 0$

$c(\infty,t) = 0 \quad @ \quad t \geq 0$

$$c(x,t) = \frac{co}{2}\left[erfc\left[\frac{x - \bar{v}t}{2(Dt)^{0.5}}\right] + \exp\left(\frac{\bar{v}x}{D}\right).erfc\left[\frac{x + \bar{v}t}{2(Dt)^{0.5}}\right]\right]$$

where:

1. Diffusion is small compared to mechanical dispersion then $D = \alpha.\bar{v}$
2. The second term of the above equation is negligible because its value is so small when dispersivity of porous medium is large or when x or t is large.

- note that erfc(∞)=0 , $e^{-\infty} = 0 \quad , \quad e^{0} = 1$

Then we get

$$\frac{c}{c_o} = \frac{1}{2}.erfc\left[\frac{x-\bar{v}t}{2(Dt)^{0.5}}\right]$$

where : erfc = complementary error function

$$erfc(B) = 1 - erf(B)$$
$$erf(-B) = -erf(B)$$
$$erfc(-B) = 1 + erf(B)$$

$$B = \frac{x-\bar{v}t}{2(\alpha\bar{v}t)^{0.5}}$$

From table (1) , B , erf(B) , erfc(B)

- If we know B we know erfc(B) , and if we know erfc (B) , we know B

Example: A non sorbing species is sent through a column (30cm) in length at a velocity of 1×10^{-2} cm/s. c/c_o ratios of 0.42 and 0.573 are noted at 46.6 and 53.3 minutes respectively , after the test started , what is the dispersivity.

solution : using the first breakthrough concentration

$$\frac{c}{c_o} = \frac{1}{2}.erfc\left[\frac{x-\bar{v}t}{2(Dt)^{0.5}}\right]$$

$$0.42 = \frac{1}{2}erfc\left[\frac{30-1\times10^{-2}\frac{cm}{s}\times46.6\min\times60\frac{s}{\min}}{2(\alpha\times1\times10^{-2}\times46.6\times60)^{0.5}}\right]$$

$$0.42 = \frac{1}{2}erfc\left[\frac{30-28}{2(\alpha\times28)^{0.5}}\right]$$

$$0.84 = erfc(B)$$

From table

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{0.84-0.887537}{x-0.1}=\frac{0.832004-0.887537}{0.15-0.1}$$

$$\frac{-0.047537}{x-0.1}=\frac{-0.055533}{0.05}$$

$$-2.37685\times10^{-3}=-0.055533X+5.5533\times10^{-3}$$

$$0.055533X=7.93015\times10^{-3}$$

$$X=0.14=B$$

Substitute in the equation

$$0.14=\frac{30-28}{2(\alpha\times28)^{0.5}}\Rightarrow0.14=\frac{2}{2\sqrt{28\alpha}}\Rightarrow\alpha=1.8$$

The second calculation is the same

$$0.573=\frac{1}{2}erfc\left[\frac{30-1\times10^{-2}\times53.3\times60}{2(\alpha\times1\times10^{-2}\times53.3\times60)^{0.5}}\right]$$

$$0.573=\frac{1}{2}erfc\left[\frac{30-32}{2\sqrt{32\alpha}}\right]$$

$$1.146=erfc\left[\frac{-2}{2\sqrt{32\alpha}}\right]$$

$$1.146=erfc(-B)=1+erf(B)$$

$$1.146=1+erf(B)$$

$$erf(B)1.146-1$$

$$=0.146$$

From table

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{0.146-0.112463}{x-0.1}=\frac{0.167996-0.112463}{0.15-0.1}$$

$$\frac{0.033537}{x-0.1}=\frac{-0.055533}{0.05}$$

$$1.67685\times10^{-3}=0.055533X-5.5533\times10^{-3}$$

$$X=0.13=B$$

$$0.13=\frac{2}{2\sqrt{32\alpha}}$$

$$\alpha=1.8$$

- ***Longitudinal dispersion*** spreading of the solute in the direction of the bulk flow
- ***Transvered dispersion*** spreading in direction perpendicular to the flow.

Instantaneous Point Source Model

The accidental spill , frequently referred as an instantaneous or pulse – type problem., such as subsurface disposal of radioactive waste or highly toxic inorganic or organic compounds.

One of the characteristic features of dispersive process ; ***it cause spreading of the solute in direction transverse to flow path as well as in the longitudinal flow direction*** , as can be seen in the following figure 1.

a. Continuous tracer feed with step function

In this experimental sand box a non reactive tracer is introduced as continuous steady state input to the uniform flow field.

Dispersion in this two – dimensional flow domain is shown below:

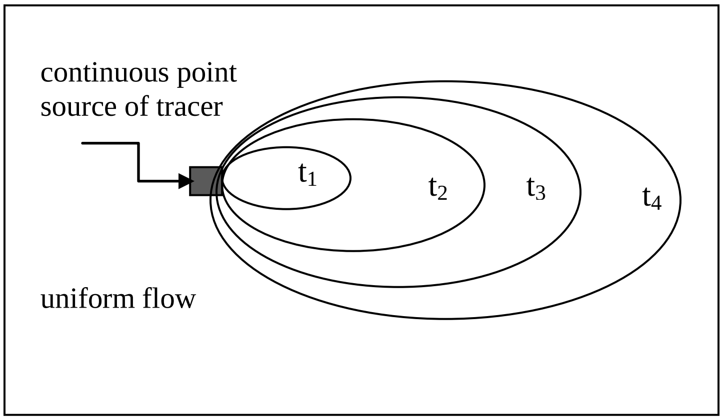


Figure 1: spreading of tracer in a two dimensional uniform flow in an isotropic sand, continuous tracer feed with step function

b. Instantaneous point source

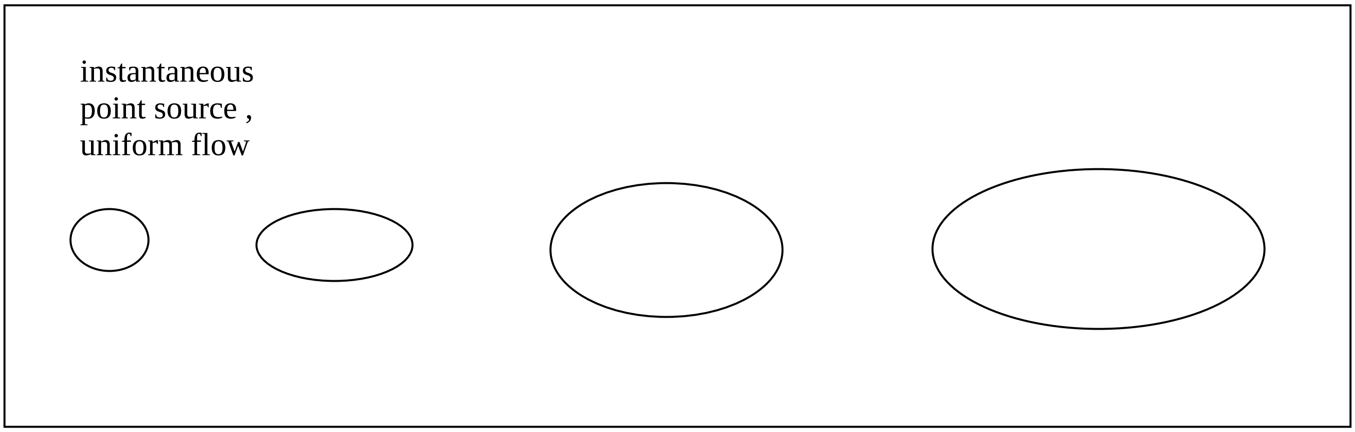


Figure 2 : Instantaneous point source

In this experiment we can see:

- The tracer spreads in all direction in the horizontal plane.
- The total mass of the tracer does not change but the mass occupies an increasing volume of the porous medium.
- The figure above shows that the tracer zone develops an elliptical shape as the tracer is transported through the system.
- Because the process of mechanical dispersion is anisotropic dispersion is stronger in the direction of flow (the longitudinal dispersion) than in direction normal to the flow line.

Baetsle, 1969, described the accidental spill as follows:
The contaminant is assumed to be originated as an instantaneous slug at a point source where x=0 , y=0 , z=0

The mass of contaminant is then carried away from the source by transport in a steady – state uniform flow field moving in the x – direction in a homogeneous isotropic medium.

As the contaminant mass is transported through the flow system, the concentration distribution of the contaminant mass at time t is given by:

$$C(x,y,z,t)=\frac{M}{8(\pi t)^{3/2}.\sqrt{Dx.Dy.Dz}}.\exp\left[\frac{-X^2}{4Dx.t}-\frac{Y^2}{4Dyt}-\frac{Z^2}{4Dz.t}\right] \qquad \text{…… (1)}$$

Where: M is the mass of contaminant introduced at the point source=$Co.Vo$

Co= the initial concentration
Vo= the initial volume
Dx , Dy , Dz = coefficient of dispersion in the x , y , z direction
X, Y , Z = distance in the x , y , z direction

We have:

$$X = x-\bar{v}t$$

$$Y = y$$

$$Z = z$$

لاننا فرضنا انه يتحرك باتجاه واحد هو x

The max. concentration is located at the center of gravity of the contaminant cloud where X=0 , Y=0 , Z=0 ; so equation(1) become

$$C_{\max} = \frac{M}{8(\pi t)^{3/2}.\sqrt{Dx.Dy.Dz}}$$

Transport of Reactive Constituents

The chemical and biological reactions that can alter concentrations in ground water flow systems can be grouped in following categories:

1. Adsorption – desorption reactions

2. Ion pairing complexation
3. Acid – base reactions
4. Solution- precipitation reactions
5. Oxidation – reduction reactions
6. Microbial cell synthesis
7. Radioactive decay

(We will focus on adsorption here)

For homogeneous saturated media with steady – state flow, the one dimensional form of the advection – dispersion eq. which include the adsorption process:

$$D\frac{\partial^2 C}{\partial x^2} - \bar{v}\frac{\partial C}{\partial x} + \frac{\rho_b}{n}\frac{\partial S}{\partial t} = \frac{\partial C}{\partial t} \quad \text{….(1)}$$

Where:

ρ_b : Bulk mass density of the porous medium
n : Porosity
S : Mass of the chemical constituent adsorbed on the solid part of the porous medium / unit mass of solids (mg/kg)
$\frac{\partial S}{\partial t}$: The rate at which the constituent is adsorbed (rate of adsorption)
$\frac{\rho_b}{n}\frac{\partial S}{\partial t}$; The change in concentration in the fluid causes by adsorption or desorption

- Adsorption reactions for contaminants in ground water are normally being very rapid relative to the flow velocity
- The amount of the contaminant that is absorbed by the solids is a function of the concentration in the solution

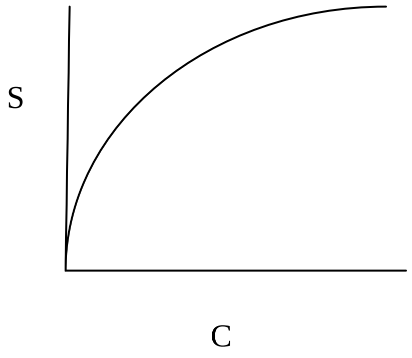
$$S = f(C)$$

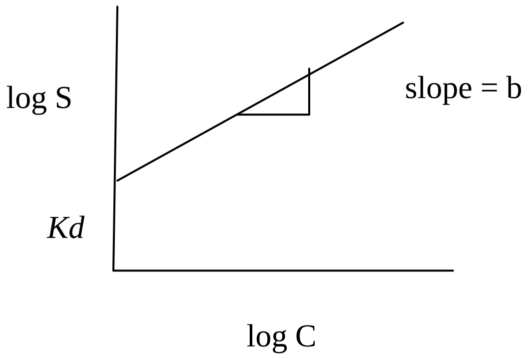
And fallows $\frac{\partial S}{\partial t} = \frac{\partial S}{\partial C}.\frac{\partial C}{\partial t}$ ……..(2)

And multiply by $\frac{\rho_b}{n}$

$$\frac{\rho_b}{n}\frac{\partial S}{\partial t} = \frac{\rho_b}{n}\frac{\partial S}{\partial C}.\frac{\partial C}{\partial t}$$

In which $\frac{\partial S}{\partial t}$: partitioning of the contaminant between the solution and the solute = k_d (distribution coefficient)





The graphical representation:

- Mass adsorbed per unit mass of dry solids is plotted against the concentration of the constituent in solution.
- This graphical representation between S and C and their equivalent mathematical expression are known as isotherms.
- This term derives from the fact that adsorption experiments are normally conducted at constant temperature.

Where $S = kdC^b$

$$\log S = \log kd + b\log C \quad \text{……..(3)}$$

This eq. is known as Frenndlich Isotherm

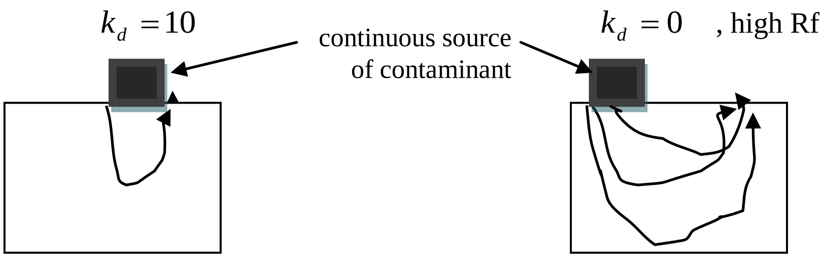
$S =$ mass of solute adsorbed or precipitated on the solids per unit bulk dry mass of the porous medium

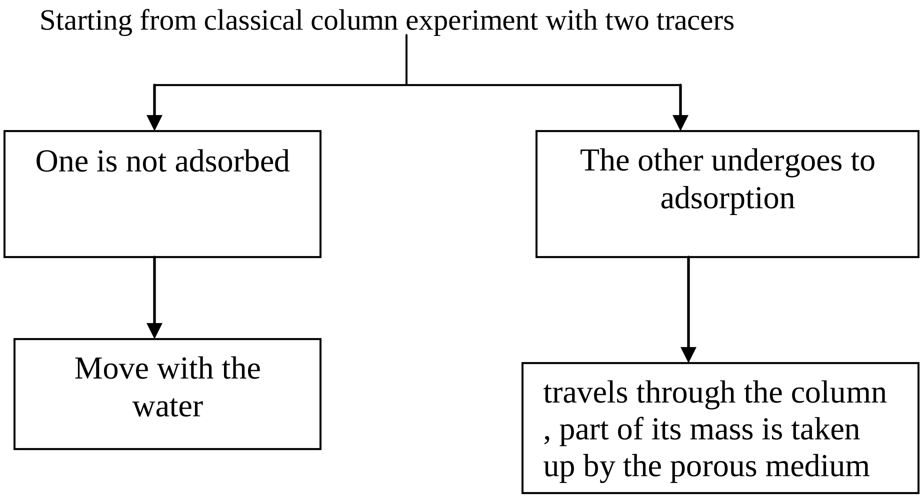
C and k_d :coefficients that depend on the solute species , nature of the porous medium and other conditions of the system.

$b =$ slope, when b=1 for linear isotherm then $\frac{\partial S}{\partial C} = k_d$

k_d (Distribution coefficient): is a valid representation of the partitioning between liquid and solids only if the reaction that cause the partitioning is fast and reversible and only if the isotherm is linear.

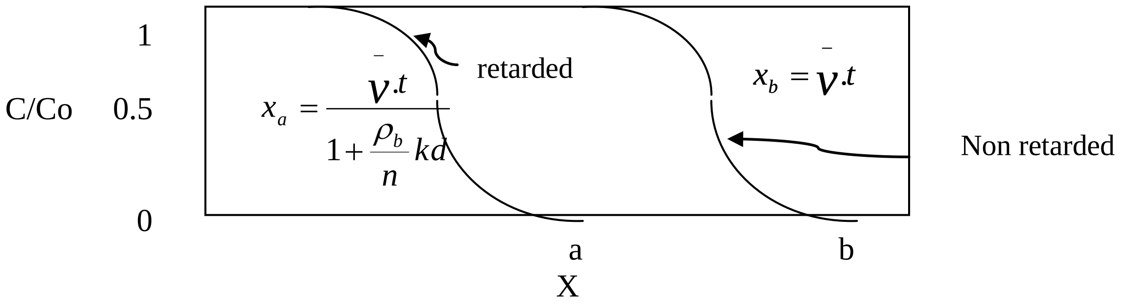
- For $k_d \succ 1$ the solute is essentially immobile





- For general case $k_{xy} = k_{xz} = k_{yx} = k_{yz} = k_{zx} = k_{zy} = 0$

- The tracers distribution in the column represented schematically.



Advanced of adsorbed and non adsorbed solute

- The non reactive tracer move ahead of the reactive tracer → speed as a result of dispersion
- The reactive tracer spread out but travels behind the non reactive tracer , therefore the adsorbed tracer is said to be retarded

$$R_f = \frac{\bar{v}}{v_c} = 1 + \frac{\rho_b}{n}.kd \qquad \text{....(4)} \qquad \text{(retardation equation)}$$

Where R_f : retardation factor

$\bar{v}$ = average linear velocity of ground water

v_c = the velocity of the C/Co point on the concentration profile of the retarded constituent

R_f can be written as : $R_f = 1 + \left(\frac{1-n}{n}\right)\rho_s.kd$

The velocity of the contaminant becomes less than the velocity of ground water.

$$v_c = \frac{\bar{v}}{R_f} = \frac{\bar{v}}{1 + \left(\frac{1-n}{n}\right).\rho_s.kd}$$

ρ_s =2.65 gm/cm^3

kd in ml/gm

the retardation equation predicts the position of the front of a plume to advection transport with adsorption described by a simple linear isotherm

$\frac{\bar{v}}{v_c}$: describes how many times faster the ground water (or non sorbing tracer) is moving relative to the contaminant being adsorbed.

If kd=0 then no adsorption

- The covering equation for mass transport through ground water can be represented as (including retardation).

$$\frac{Dx}{R_f}.\frac{\partial^2 C}{\partial x^2} - \frac{v_x}{R_f}.\frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$

- Solution of this equation with the same boundary condition used with Ogata – Bank (1961) eq. is

$$\frac{C}{Co} = \frac{1}{2}.erfc\left[\frac{R_f.x - \bar{v}.t}{2\left(\alpha.\bar{v}.t.R_f\right)^{0.5}}\right]$$

Radioactive Decay , Biodegradation and Hydrolysis

Consider mass transport involving a first order kinetic reaction

$$D_x.\frac{\partial^2 C}{\partial x^2} - v_x.\frac{\partial C}{\partial x} - \lambda C = \frac{\partial C}{\partial t}$$

Where λ : the decay constant for radioactive decay and it equal to $\frac{0.693}{t^{1/2}}$

$t^{1/2}$; Half life time

- The same initial and boundary conditions of the Ogata – Bank equation in one dimension is

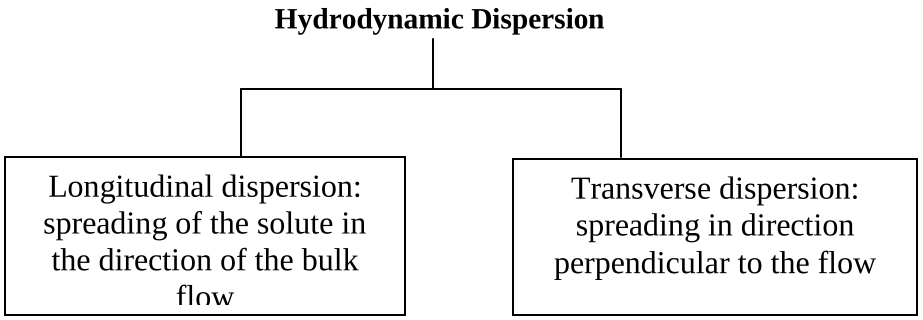
$$\frac{C}{Co} = \frac{1}{2}.\exp\left[\left(\frac{x}{2\alpha_x}\right)\left(1-\left(1+\frac{4\lambda\alpha_x}{v}\right)^{0.5}\right)\right].erfc\left[\frac{.x - v.t\left(1+ {4\lambda\alpha_x}/{v}\right)^{0.5}}{2(\alpha.v_x.t)^{0.5}}\right] \quad \ldots(1)$$

Where v is the contaminate velocity and its equal v_w/R_f

That is mean the important of retardation in problems of decay or degradation

If λ =0 eq. (1) reduced to Ogata – Bank eq.

If λ are large → exp term reach zero → concentration approach zero → the materials decaying faster than it can be transported through the system



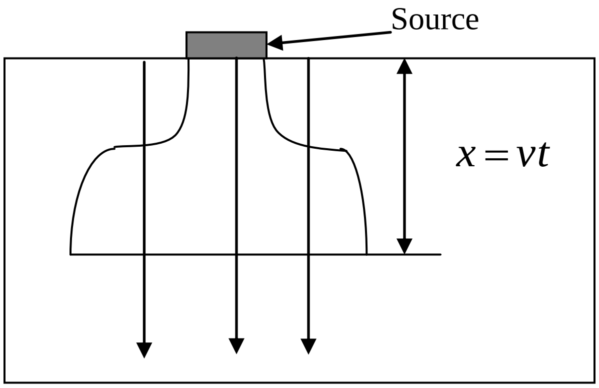
Hydrodynamic Dispersion

Longitudinal dispersion: spreading of the solute in the direction of the bulk flow

Transverse dispersion: spreading in direction perpendicular to the flow

Transverse dispersion:

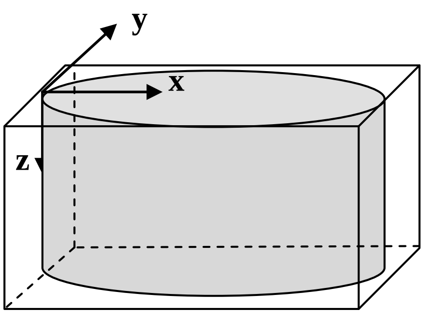
The flowing figure illustrate an advective model with transverse dispersion (lateral dispersion) in the absence of longitudinal dispersion



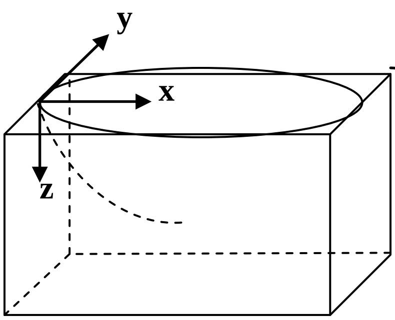
- Transverse dispersion depend on the source geometry

Geometric source configuration includes:

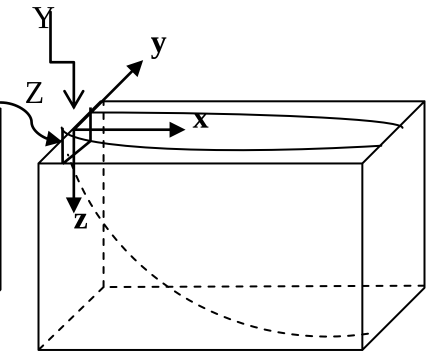
- Vertical line source
- Point source
- Plane source ,(is more practical)



a. Vertical line source



b. point source



c. plane source

- For figure (a):Large σ_Y , the greater the lateral extent of the plume
- For figure (b) : lateral σ_Y and vertical σ_Z spreading
- For figure (c):lateral σ_Y and vertical σ_Z spreading (as well as in the x-direction) vertical spreading downward σ_X .

The equations describing the parts of a plum controlled by transverse spreading for a point or line source are complicated, so; the equations that fallow are limited to describe the maximum concentration along the plane of symmetry , along the x- axis for y =0 and z=0 .

$$C_{\max} = C_o$$ Pulg flow

$$C_{\max} = \frac{C_o.Q}{2v(\pi.\sigma_Y x)^{1/2}}$$ Line source

$$C_{\max} = \frac{C_o.Q}{4x\pi v(\sigma_Y \sigma_Z)^{1/2}}$$ Point source

Where Q; volumetric flow rate (L^2/T) units for line source and (L^3/T) for point source

- Domenico and Palciauskas (1982) developed a solution for the more practical dispersion , the plane source as followes:

$$C = \left(\frac{C}{2}\right)\left(\left[erf\ \frac{y+\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right] - erf\left[\frac{y-\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right]\right) \qquad (1)$$

Where The half source size Y/2 is part of the solution.
the z component of spreading is:

$$C = \left(\frac{C}{2}\right)\left(\left[erf\ \frac{z+Z}{2(\alpha_z x)^{1/2}}\right] - erf\left[\frac{z-(Z)}{2(\alpha_z x)^{1/2}}\right]\right) \qquad (2)$$

Where the full source size Z becomes part of the solution

For spreading directions in both y and z , we get

$$C = \left(\frac{C}{4}\right)\left(\left[erf\ \frac{y+\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right] - erf\left[\frac{y-\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right]\right)\left(\left[erf\ \frac{z+Z}{2(\alpha_z x)^{1/2}}\right] - erf\left[\frac{z-(Z)}{2(\alpha_z x)^{1/2}}\right]\right)$$

For plane of symmetry (y=z=0)

$$C_{\max} = (C_o)\left(\left[erf\ \frac{Y}{4(\alpha_y x)^{1/2}}\right].erf\left[\frac{Z}{2(\alpha_z x)^{1/2}}\right]\right)$$

Models for multidimensional transport

Multidimensional transport involves both longitudinal and transverse dispersion in addition to advection

$$D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - v_x \frac{\partial C}{\partial x} - \frac{r}{n} = \frac{\partial C}{\partial t}$$

Where r is defined by some mathematical rate law

Continuous Source

Models that include transverse spreading most incorporate information on the source geometry

$$C_{(x,y,z,t)} = \left(\frac{C_o}{8}\right)\left[efr\,\frac{(x-vt)}{2(\alpha_x vt)^{1/2}}\right]\left(\left[erf\,\frac{y+\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right]-erf\left[\frac{y-\left(Y/2\right)}{2(\alpha_y x)^{1/2}}\right]\right)\left(\left[erf\,\frac{z+Z}{2(\alpha_z x)^{1/2}}\right]-erf\left[\frac{z-(Z)}{2(\alpha_z x)^{1/2}}\right]\right)$$

For plane of symmetry y=z=0, the above equation become

$$C_{(x,0,0,t)} = \left(\frac{C_o}{2}\right)\left(\left[erfc\,\frac{(x-vt)}{2(\alpha_x vt)^{1/2}}\right]\left[erf\,\frac{Y}{4(\alpha_y x)^{1/2}}\right].erf\left[\frac{Z}{2(\alpha_z x)^{1/2}}\right]\right)$$

Example : Drums of diethyl ether (de) and carbon tetrachloride (ct) were buried in a sand aquifer 15 years ago . Calculate the concentration of each contaminant along the plane of symmetry of the plume at the point (x=225m, y=0, z=0) at time 15 years (4.73×10^{8}s) , the velocity of the ground water is (1×10^{-6}m/sec) . the retardation factor for de is 1.5 and for ct 27.4 , the source concentration for de is (1×10^{4}µg/l) and for ct is (5×10^{2}µg/l). The source size in Y is 25m and in Z is 5m, the estimated dispersivities are α_x=1 ,α_y=0.1m and α_z=0.01m

$$v = \frac{v_w}{R_f}$$

$$C(x,0,0,t) = \left(\frac{C_o}{2}\right) erfc\left[\frac{x - vt}{2(\alpha_x vt)^{0.5}}\right].erf\left[\frac{Y}{4(\alpha_y x)^{0.5}}\right].erf\left[\frac{Z}{2(\alpha_z x)^{0.5}}\right]$$

For de

$$C = \left(\frac{1\times10^{4}}{2}\right).erfc\left[\frac{225 - \left(1\times10^{-6}/1.5\right)\left(4.73\times10^{8}\right)}{2\left(\frac{1\times10^{-6}}{1.5}\times4.73\times10^{8}\right)^{0.5}}\right] erf\left[\frac{25}{4(0.1\times225)^{0.5}}\right].erf\left[\frac{5}{2(0.01\times225)^{0.5}}\right]$$

$$= \left(5\times10^{3}\right) erfc\left[\frac{-90.33}{35.515}\right] erf\,(1.3176)\, erf\,(1.6667)$$

$$erfc(-\beta) = 1 + erf\,(\beta)$$

$$\therefore\ erfc(-2.543) = 1 + erf\,(2.543)$$

$*$ *from table*

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 0.999593}{2.543 - 2.5} = \frac{0.999764 - 0.999593}{2.6 - 2.5} \Rightarrow y = 0.999666 = erf\,(\beta)$$

$$\therefore\ erfc(\beta) = 1 + 0.999666 = 1.999666$$

$* erf\,(1.3176) \Rightarrow$ *from table*

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y - 0.934008}{1.3176 - 1.3} = \frac{0.952285 - 0.934008}{1.4 - 1.3} \Rightarrow y = 0.93722 = erf\,(\beta)$$

$* erf\,(1.66667)$ *from table*

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y - 0.976348}{1.6667 - 1.6} = \frac{0.983790 - 0.976348}{1.7 - 1.6} \Rightarrow y = 0.98131$$

$$c_{de} = 5000\times1.999666\times0.93722\times0.98131 = 9195.498\,\mu g/l$$

This is the max. concentration for de

- For Ct

$$C_{(x,0,0,t)}=\left(\frac{C_o}{2}\right)\left(\left[erfc\frac{(x-vt)}{2(\alpha_x vt)^{1/2}}\right]\left[erf\frac{Y}{4(\alpha_y x)^{1/2}}\right].erf\left[\frac{Z}{2(\alpha_z x)^{1/2}}\right]\right)$$

$$=\left(\frac{5\times10^2}{2}\right)erfc\left[\frac{225-(1\times10^{-6}/27.4)(4.73\times10^8)}{2\left(1\times\frac{1\times10^{-6}}{27.4}\times4.73\times10^8\right)^{0.5}}\right]erf\left[\frac{25}{4(0.1\times225)^{0.5}}\right]erf\left[\frac{5}{2(0.01\times225)^{0.5}}\right]$$

$$=250erfc(25)erf(1.3176)erf(1.6667)$$

Because the erf(25)≈0, it follows that carbon tetreachloride has not yet reached this point because of greater retardation

The maximum concentration of ct will attain is

$$C_{\max}=C_o\left(\left[erf\frac{Y}{4(\alpha_y x)^{1/2}}\right].erf\left[\frac{Z}{2(\alpha_z x)^{1/2}}\right]\right)$$

$$=5\times10^2 erf(1.3176)erf(1.6667)=459.85\mu g/l$$

Measurement of Parameters for Ground Water

1. ***Velocity determination***: there are three groups of methods for determinate the velocity

   a. Darcy equation : Includes all techniques that are directly dependent on use of Darcy equation

      Disadvantage : this method have large uncertainties because it based on use of parameters in Darcy equation (k , gradient , and porosity)

   b. Involves the use of artificial tracers
      Involve introduce a tracer at one point in the flow field and observing its arrival at other points. The ground water velocity can be computed from the travel time and distance data.

      Disadvantage:

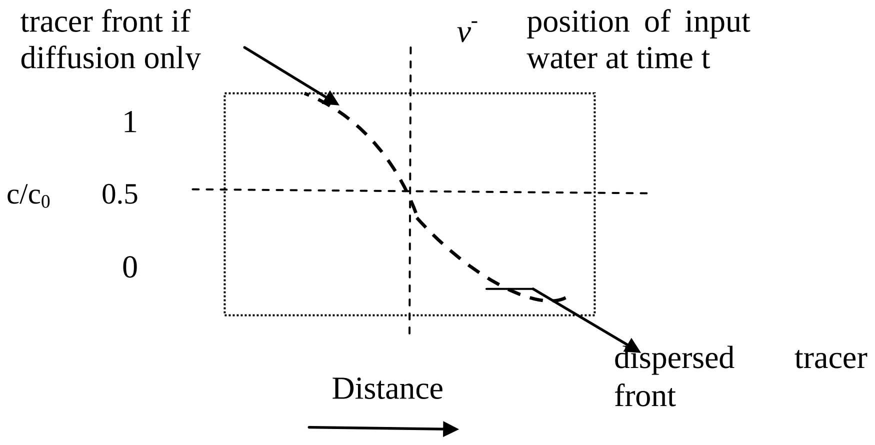
- because the ground water velocity are rarely large under natural conditions so long period of time are normally required for tracers to move significant distances through the flow system
- Geological materials are typically quite heterogeneous

   c. Ground water age – dating method using environmental isotopes such as tritium and carbon 14, which can be accurately monitored using radioactivity detectors

      Disadvantage: this method need government licensing requirements for their use and hazardous when used by careless workers.

2. ***Dispersivity*** : the most elusive of the solute transport parameters ; there are four method for measuring it :

a. Laboratory experiments: longitudinal dispersivity can be measured in the laboratory by passing a non reactive tracer through cylindrical samples collected from boreholes or excavation , these experiments procedure break – through curve



The dispersivity of the sample can be computed by fitting solution of the advection – dispersion equation to the experimentally determined break – through curve.

b. Field method: there are four types of field dispersivity tests:

(1) Single – well withdrawal test injection

The tracer is pumped in a set time period followed by pumping fom the well and monitoring the concentration travel with time.

(2) Natural – gradient test

The tracer is introduced into the system , its migration is then monitored at one or more sampling points.

(3) Two well recirculation test

The tracer is injected into the flow regime at one well , it is pumped out the second well and then recirculated through the withdrawal injection system.
The concentration versus time response at the withdrawal well serves as a basis for computation of the dispersivity using analytical or numerical models .

(4) Two well pulse test

Tracer is introduced into a well situated within the drawdown concentration caused by pumping of the second well ; concentration data from pumping well are used for calculation of dispersivity value for the segment of the formation between the two wells.

c. From table and charts
d. From similar case studies

Remediation

There are four main alternatives for dealing with problems of contamination:

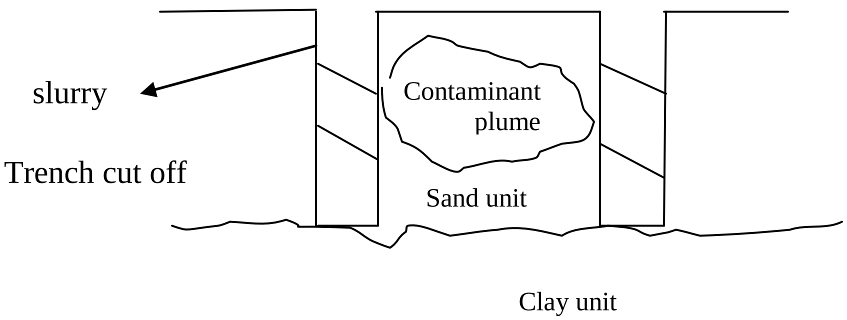
1. Containing the contaminants in place
2. Removing contaminants from the ground altogether
3. Treating the contaminants in situ
4. Attenuating the possible hazard by institutional control

Containment

A series of control measures that keep the contaminants in the ground prevent further spread through the use of physical or hydrodynamic barriers such as:

a. Slurry walls (slurry trench cutoff wells):

Are low permeability barriers →they confine contaminants by either surrounding the spill or by removing the potential for flow through the source with upstream barriers



Slurry wells constructed (or dug) $\xrightarrow{\text{Through}}$ Bentonite slurry $\xrightarrow{\text{Slurry well is soldified}}$ By adding cement

- A typical slurry wall ranges in width 0.5-2m and can be installed to depth of up to about 50m

b. Sheet pile cutoff wells:

Interlocking steel piles into the ground

↓ Because of the joints between piles

Piles wells typically leak, leakage can be reduced through as fines fill the joints

Disadvantage : large costs associated with construction a pile well.

c. Grouting (Grout curtain):

Grouting by injecting fluids under pressure into the ground

↓

The grouting material move away from the zone of injection and solidify thereby reducing the hydraulic conductivity and fill the voids between the grains

The grouts are :

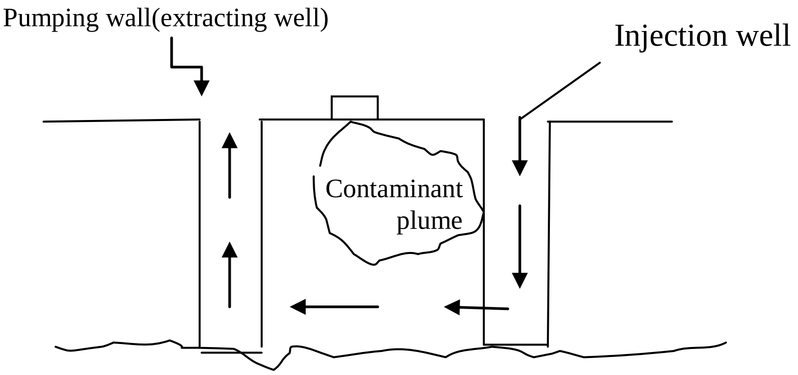
- Cement
- Bentonite
- Silicate

d. Geomembranes :

Synthetic sheets installed in open tranches to control contaminant spread (in research on development stage)

e. Hydrodynamic control:

- Lower the water table to prevent discharge to rivers or lake by pumping / injection wells
- Using wall upstream and downstream of the spill.
- حفر بئر قبل الplume وبعدها نرى اتجاهها فنضع الماء فنعكس تجاه الضخ من البئر الثاني



Contamination withdrawal

There are four different approaches:

1. Pumping : using wells to remove contaminants from the ground
2. Interceptor system : use drains , tranches and lined tranches to collect contaminants close to the water table

- Efficient at removing shallow contamination

3. Soil venting: remove volatile organic compounds from the unsaturated zone by vacuum pumping
4. Excavation: its exercise in digging and trucking.

- The greatest problems are the costs involved and finding an appropriate place to dispose of the contaminated soil.

In situ treatment of contaminants

1. Biological degradation :

Using organic contaminants as an energy source for bacteria and producing simple compounds like water and CO_2 from complex organic molecules and adding nutrients like nitrogen and phosphorous.

2. Chemical degradation:

By injecting an appropriate chemical or treatment agent.

The agent has to be specific to particular classes of contaminants, for example ; addition of alkalis or sulfaides can cause heavy metals to precipitates as insoluble minerals.

Steps for dealing with problems of contaminants in ground water

Starting with field slurry

- Site characteristics
  a. Aquifer properties
    1. Permeability
    2. Specific yield
    3. Clay content of soil
    4. Heterogeneity of formation
    5. Depth
    6. Thickness
    7. Direction of ground water
    8. Reachrge and discharge
    9. Seasonal fluctuation in water table
  b. Ground water quality
    1. Inorganic nutrient levels
    2. Precipitation of inorganic nutrients
    3. Dissolved oxygen content
  c. Contaminant characteristics
    1. Type
    2. Concentration
    3. Areal and vertical extent of contamination
    4. Location of released material in aquifer (dissolved , floating , trapped , or sinking)
    5. Heterogeneity of contamination
    6. Biodegradability of contaminants
    7. Presence and quantity of toxic agent
    8. Nature of release –acute , chronic or periodic
    9. Time since release
    10. Effect of physical weathering or a biotic reaction on contaminants
  d. Microbial characteristics
    1. Presence of active microbial population
    2. Acclimation to contaminant
    3. Nutrient requirements for optimal growth
    4. Extent of biodegradation that can be achieved
    5. Rating biodegradation